Application of Fourier Series to Determine the Measurements Error of Harmonics with Selected Power Quality Analyzers

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Abstract. *The paper presents examples of the results of experimental research concerning the influence of input signal RMS on the measurement error of harmonics using selected power quality analyzers. The experiments were carried out in a designed and prepared measuring system. The system makes it possible to set selected testing signals. In the case considered, the input signals were, respectively, sinusoidal signal, triangle signal, rectangular signal and full-wave rectification signal. Based on the expansion of selected functions in the trigonometric Fourier series, the RMS values of consecutive harmonics were obtained, which became the reference values. Metrological interpretation of the obtained results, analysis of errors, and formulation of conclusions were presented in the paper.*

Keywords: Fourier Series, Harmonics, Power Quality Analysers

1. Introduction

Intense development of technology brought the application of a number of receivers generating distorted waveforms, of voltage or current, in the power network. Therefore, it is necessary to analyze this type of signals. Such analysis generally applies the trigonometric Fourier series, which enables an arbitrary periodic function that meets the Dirichlet conditions to be presented as a sum of trigonometric functions.

A set of quantities evaluating distortion waveforms contains the measures of harmonics content. The harmonic component we call each component with a frequency being an integer multiple of the basic frequency.

Harmonics can be characterized:

- individually, by their relative amplitude u_h compared to the voltage of basic component U_1, where h is the harmonic order,
- jointly, by the Total Harmonic Distortion factor of supply voltage THD calculated according to dependence (1):

$$THD = \sqrt{\sum_{h=2}^{\infty} (u_h)^2} \tag{1}$$

In practice, to analyze the signals applied to evaluate power quality, measurement instruments called power quality analyzers are used. Related problems were discussed in a lot of publications, among others in [1] and [2].

In this paper, we present exemplary results of experimental research on the influence of RMS of the input signal on the measurement error of harmonics content with selected power quality analyzers. The measurements were conducted in a designed and prepared measuring system. A few selected periodic functions were expanded in the Fourier series for the purpose of experiments.

2. Measurements of Harmonics

According to the theorem on the expansion of a function in the Fourier series, each periodic function meeting the Dirichlet criterion can be presented according to dependence (2):

$$f(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t), \tag{2}$$

where: $\omega_0 = \frac{2\pi}{T}$, T is the period of investigated function, and a_0, a_n, b_n are successive Euler-Fourier coefficients.

To determine the measurement error of harmonics, it is necessary to know the reference value of testing functions. To this end, the functions applied in the research were expanded in the Fourier series: sinusoidal waveform containing only one harmonic, the triangle waveform, rectangular waveform, and full-wave rectification signal.

3. Experimental Research

The experiments were carried out in a measuring system whose block diagram is presented in Fig. 1. The system contains a source of testing signals of periodic functions of voltage $u(t)$, investigated power quality analyzers HA 2000 and PQ-Box 100, and a PC computer. As the source of testing signals, we used Agilent 3320A. In order to generate signals with RMS equal to 230 V, Chroma 61502 generator was used as an amplifier.

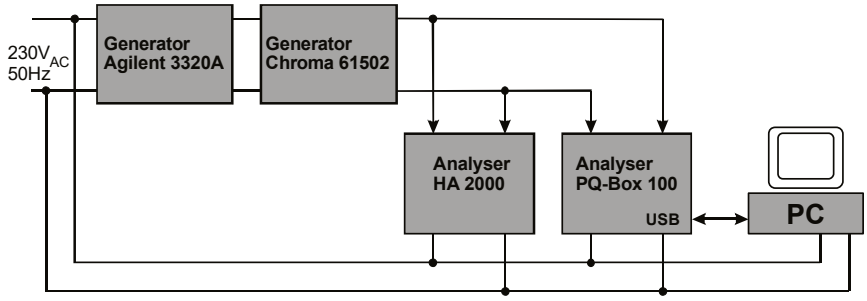

Fig. 1. Block diagram of the measuring system.

For testing signals different from sinusoidal signals, the error values δ were determined according to dependence (3):

$$\delta = \frac{|U_a - U_F|}{U_F} \cdot 100, \tag{3}$$

where U_a means the RMS of particular harmonics determined by the analyzer according to documents [3-5], and U_F is the RMS of particular harmonics obtained from the expansion of the function in the Fourier series. In order to evaluate the value of this error for the sinusoidal signal, the dependence for the Total Harmonic Distortion factor THD, given with formula (1), was used. For ideal sinusoidal signal, the value of this factor should be equal to zero. The determined values of THD, different from zero, are the error measure for a given measurement instrument.

The first testing signal was a sinusoidal signal with network frequency equal to 50 Hz for RMS equal successively to: 10 V, 50 V, 100 V, 150 V, 230 V. Due to differentiability and continuity, the sinusoidal signal is in common use in modern measuring technique. Thanks to the features listed above, such signal can be reproduced with high accuracy. Moreover, such signal has only the basic component, which enables a measuring system to be analyzed in a unequivocal way. The measurement results are presented in Fig. 2.

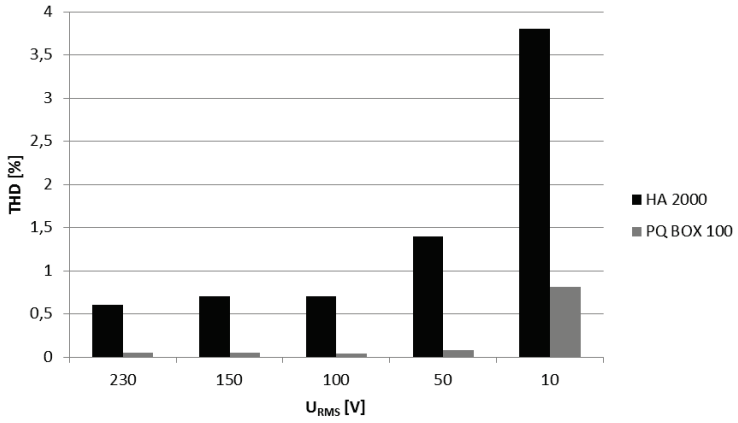

Fig. 2. Distribution of THD in dependence of signal on RMS of sinusoidal signal.

As it can be observed, the values of THD factor, being for this signal the error measure, differ essentially for both analyzers. The THD values for PQ Box 100 analyzer, apart from the measurement for voltage 10 V, are of the order of the hundredths of a percent. In the case of HA 2000 analyzer, the values of this factor amount to values of the order of single percents. Moreover, a dependence of the increase in the value of error δ on the decrease in the RMS of the input signal is revealed.

Another analyzed signals were: triangle waveform, the rectangular waveform, and full-wave rectification waveform. The analysis of presented dependencies of the error measurements of harmonics for all the considered waveforms is beyond of the scope of this publication. The dependencies will be discussed during the conference presentation. Fig. 3 and Fig. 4 show the dependencies of relative error values δ in the function of the order of harmonics for the rectangular waveform. Even harmonics are omitted in the diagrams because they are approximately equal to zero.

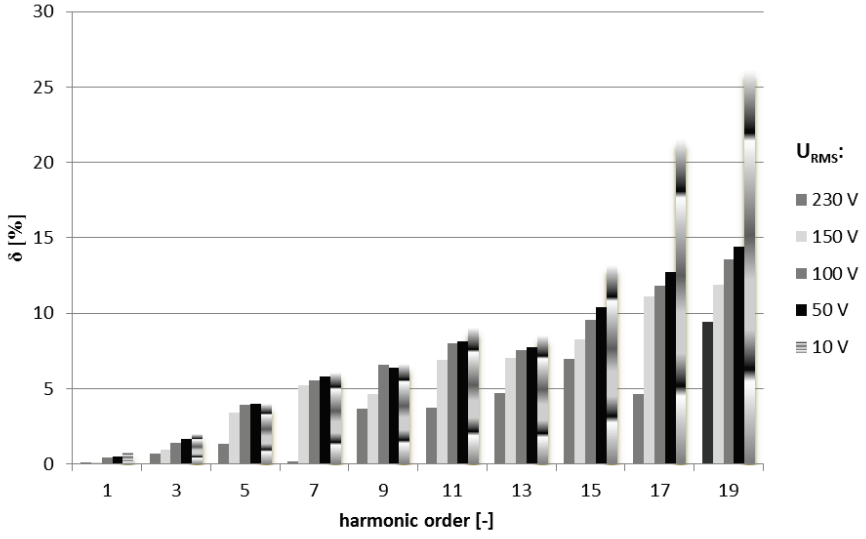

Fig. 3. Distribution of error values for rectangular waveform with use of HA 2000 analyser

4. Conclusions

The paper presents the results of experimental research concerning the influence of input signal RMS on the measurement error of harmonics. The authors measured the harmonics of

selected signals – their function description is known, and consequently, their expansion in the trigonometric Fourier series is also known. It means that the accurate values of these signals are known. The obtained results enabled the authors to determine the error values between the values obtained as in measurements and the values determined on the basis of the expansion of functions in the Fourier series.

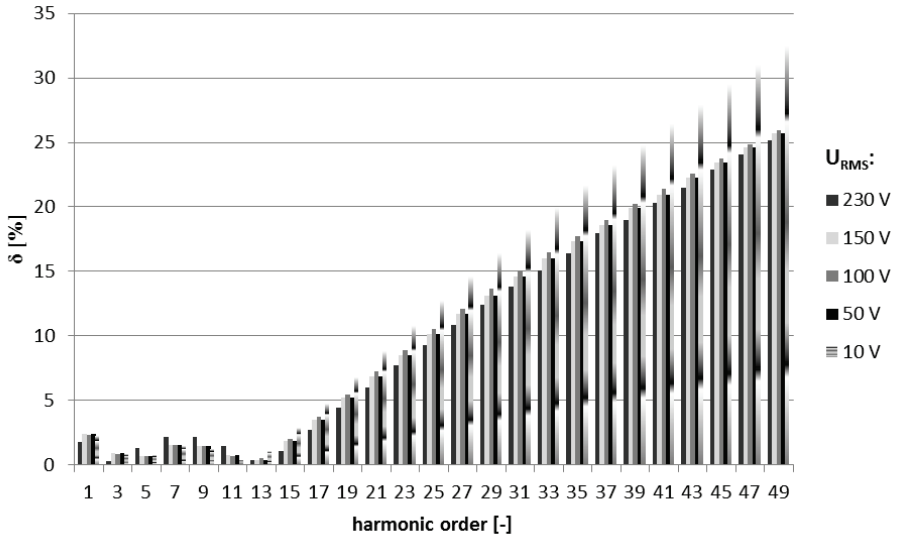

Fig. 4. Distribution of error values for rectangular waveform with use of PQ Box 100 analyzer

Based on the obtained measurement results, we can draw a conclusion that the error values for PQ Box 100 analyzer are significantly smaller than for HA 2000. It results, most probably, from the fact that in the case of PQ Box 100 analyzer, an A/C converter with higher resolution was used. It can be noticed that together with an increase in harmonics order, the error value for both investigated analyzers increases significantly. This is particularly visible for triangle waveform and full-wave rectification waveform, for which successive harmonics decrease proportionally to the inverse of the square of their order. The analysis makes it also possible to draw a conclusion that there is an inversely proportional dependence between the RMS of the voltage signal and the measurement error of particular harmonics.

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