Controller Design for Cuk Converter Using Model Order Reduction

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***Abstract*: Cuk converter contain two inductor and two capacitor hence it is fourth order dc-dc converter. It provides output voltage both higher as well as lower than the input voltage. Design of feedback compensator for fourth order system is quite complex. In this paper, model order reduction technique is used for controller design of Cuk converter. First small signal dynamic model for Cuk converter using state space analysis (SSA) is obtained which provides fourth order transfer function. Then this fourth order transfer function is reduced to second order using Pade approximation**.

***Index Terms*- Cuk Converter, Model Order Reduction, Compensator, State-Space Averaging.**

I. Introduction

The Ćuk converter is a type of dc-dc converter that has an output voltage magnitude that is either greater than or less than the input voltage. Cuk converters has excellent properties like capacitive energy transfer, full transformer utilization and good steady-state performances such as wide conversion ratio, smooth input and output currents.

The dynamic response, however, is affected by the fourth order characteristic, which generally calls for closed-loop bandwidth limitations in order to ensure large-signal stabilization. Moreover, stability may require big energy transfer capacitors in order to decouple input and output stages.

For the purpose of optimizing the converter dynamics, while ensuring correct operation in any working condition, robust multivariable controllers could be used. These, however, may involve considerable complexity of both theoretical analysis and control implementation. To remove this difficulty first we reduce the order of transfer function of Cuk converter then design controller.

The Cuk converter is made up of two capacitors, two inductors, a power switch and diode thus it is fourth order non-linear system. For the feedback control design linear model is needed. The linear model of the converter is derived by the replacement of switch and diode of converter by small signal averaged switch model [7]. The desired transfer function is obtained using state space averaging technique. This paper presents Cuk converter operating in continuous conduction mode (CCM). In continuous conduction mode inductor current never falls to zero during one switching period. The state space averaging technique (SSA) [3] is used to find small signal linear model and its various forms of transfer functions. Depending on control-to-output transfer function, the PWM feedback controller [10] is designed to regulate the output voltage of the Cuk converter. This transfer function is found to have two pair of complex pole in left half plane and three zeros in RHP. RHP zero is undesirable for controller designing because it provides extra 90^0 phase lag. Also, higher order system increases controller complexity , to remove these difficulties model order reduction technique is used. In this paper Pade approximation [13] method is used for model order reduction. The reduced order system has one RHP zero and one pair of complex pole.

II. SSA Technique

The power stage of closed loop system is a non-linear system. Since non-linear system is difficult to model and their behavior is also difficult to predict, therefore, it is common practice to approximate non-linear system to a linear system. For linearized power stage of dc-dc converter Bode plot can be used to determine suitable compensation in feedback loop for desired steady state and transient response. For this state space averaging technique is used.

In dc-dc converter operating in continuous condition mode, there exit two states one when switch is on and other when switch is off.

During switch on;

$$\dot{X} = A_1 X + B_1 V_d \qquad 0< t < dT$$

During switch off;

$$\dot{X} = A_2 X + B_2 V_d \qquad 0< t < (1-d)T$$

$$V_0 = C_1 X \qquad \text{during } dT$$

$$V_0 = C_2 X \qquad \text{during}(1\text{-d})\ T$$

To produce an average description of the circuit over a switching period, the equations corresponding to the two foregoing states are time weighted and averaged, resulting in the following equations-

$$\dot{X} = [A_1 d + A_2(1-d)]X + [B_1 d + B_2(1-d)]V_d$$

$$V_0 = [C_1 d + C_2(1-d)]X$$

III. System Analysis

Cuk converter is a switching regulator which yields a variable output voltage from a constant dc supply. The state-space averaged model used to derive the steady-state and the dynamic models of the Cuk converter based on its state space averaged model [3].

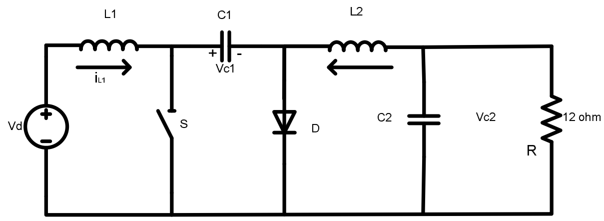

Fig.1.a Cuk converter

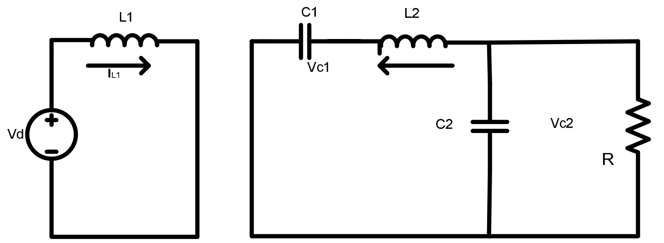

Fig.1.b Cuk converter when switch is ON

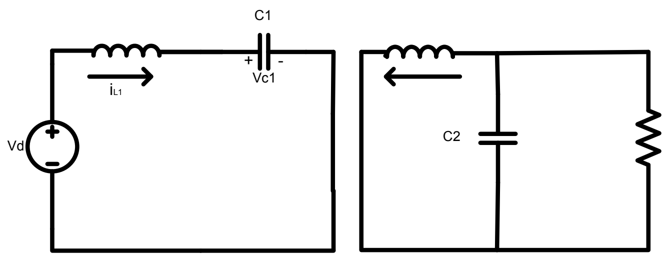

Fig.1.c Cuk converter when switch is OFF

Fig.1 Operation of Cuk converter in CCM

A. *Modeling of Cuk converter by state space technique*

The Cuk converter contains two inductors L_1 and L_2 with equivalent series resistances r_{L1}, r_{L2} respectively, two capacitors C_1 and C_2 with equivalent series resistances r_{C1} and r_{C2} respectively, switch (MOSFET) S and a diode D as shown in fig.1a. The resistance R is representing the load. The input voltage V_d is fed into the circuit via inductor L1. When switch is on as shown in fig 1.b, current i_{L1} builds the magnetic field of the inductor in the input stage. The diode D is reverse biased, and the energy dissipates from the storage elements in the output stage. When the switch is turned-off as shown in fig.1.c, the inductor L1 tries to maintain the current flowing through it by reversing polarity and sourcing current as its magnetic field collapses. It thus provides energy to the output stage of the circuit via capacitor C1. Sum of both currents i_{L1} and i_{L2} must be zero in the steady state, with the assumption that voltage v_{C1} is essentially constant (given that the voltage across a capacitor cannot change instantaneously and the switching speed of the circuit is high). This provide fallowing energy conservation relation-

$$\frac{V_o}{V_d} = \frac{d}{1-d}$$

Where d is the duty cycle of the switch. This equation shows that by controlling the duty cycle of the switch output voltage Vo can be controlled and output voltage can be higher or lower than the input voltage V_d. By using a controller to vary the duty cycle during operation, the circuit can also be made to reject disturbances.

B .*State space equation of Cuk converter*

The state space equation for Cuk converter during switch on and off are

During switch is ON

$$\frac{di_{L1}}{dt} = -\frac{r_{L1}i_{L1}}{L_1} + \frac{V_d}{L_1} \tag{1}$$

$$\frac{di_{L2}}{dt} = \frac{V_{C1}}{L_2} - \frac{(r_{L2}+r_{C1}+r_{C2})||i_{L2}}{L_2} - \left(\frac{r_{C2}}{R+r_{C2}}-1\right)\frac{V_{C2}}{C_2} \tag{2}$$

$$\frac{dv_{C1}}{dt} = -\frac{i_{L2}}{C_1} \tag{3}$$

$$\frac{dv_{C2}}{dt} = -\frac{Ri_{L2}}{(r_{C2}+R)C_2} - \frac{V_{C2}}{(R+r_{C2})C_2} \tag{4}$$

During switch off

$$\frac{di_{L1}}{dt} = \frac{(r_{L1}+r_{C1})i_{L1}}{L_1} - \frac{V_{C1}}{L_1} + \frac{V_d}{L_1} \tag{5}$$

$$\frac{di_{L2}}{dt} = \frac{(r_{L2}+R\,||\,r_{C2})i_{C2}}{L_2} + \frac{\left(\frac{r_{C2}}{R+r_{C2}}-1\right)V_{C2}}{L_2} \tag{6}$$

$$\frac{dV_{C1}}{dt} = \frac{i_{L1}}{C_1} \tag{7}$$

$$\frac{dV_{C2}}{dt} = \frac{Ri_{L2}}{(r_{C2}+R)C_2} - \frac{V_{C2}}{(R+r_{C2})C_2} \tag{8}$$

The averaged matrices for the steady-state and linear small-signal state-space equations can be written according to above equations.

$$A_1 = \begin{pmatrix} \frac{-r_{L1}}{L_1} & 0 & 0 & 0 \\ 0 & \frac{-(r_{L2}+r_{C1}+r_{C2}\,||\,R)}{L_2} & \frac{1}{L_2} & \frac{-\left(\frac{r_{C2}}{R+r_{C2}}-1\right)}{C_2} \\ \frac{1}{C_1} & 0 & 0 & 0 \\ 0 & \frac{R}{(r_{C2}+R)C_2} & 0 & \frac{-1}{(r_{C2}+R)C_2} \end{pmatrix} \tag{9}$$

$$A_2 = \begin{pmatrix} \frac{-r_{L1}+r_{C1}}{L_1} & 0 & \frac{-1}{L_2} & 0 \\ 0 & \frac{-(r_{L2}+r_{C2}\,||\,R)}{L_2} & 0 & \frac{\left(\frac{r_{C2}}{R+r_{C2}}-1\right)}{C_2} \\ \frac{1}{C_1} & 0 & 0 & 0 \\ 0 & \frac{R}{(r_{C2}+R)C_2} & 0 & \frac{-1}{(r_{C2}+R)C_2} \end{pmatrix} \tag{10}$$

$$B_1 = B_2 = B = \begin{pmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad C_1 = C_2 = C = \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}$$

$$E_1 = E_2 = E = (0)$$

C. Transfer function

With the state space matrices defined above, the control -to - output transfer function can be calculated as

$$G_{vd} = C(SI - A)^{-1} B_d + E_d \qquad (11)$$

Where $B_d = (A_1 - A_2)X + (B_1 - B_2)V_d \qquad (12)$

Output to input transfer function

$$G_{vg} = C(SI - A)^{-1} B \qquad (13)$$

$$X = -CA^{-1} V_d \qquad (14)$$

IV. CONTROL OF CUK CONVERTER

A. PWM feedback control

Fig. 2(a) shows a Cuk converter with PWM feedback control [8]. The output voltage V_0, is fed back and compared with the reference voltage, Vref. Which produces error voltage V_e which is applied to the compensator, $G_c(s)$, which produces the control voltage, V_c, to compare with the saw tooth voltage of amplitude V_M at the PWM comparator. As depicted in Fig. 2(b), the MOSFET is turned on when V_c is larger than V_{saw}, and turned off when V_c is smaller than V_{saw}. If V_0 is changed, feedback control will respond by adjusting Vc and then duty cycle of the MOSFET until V_0 is again equal to Vref

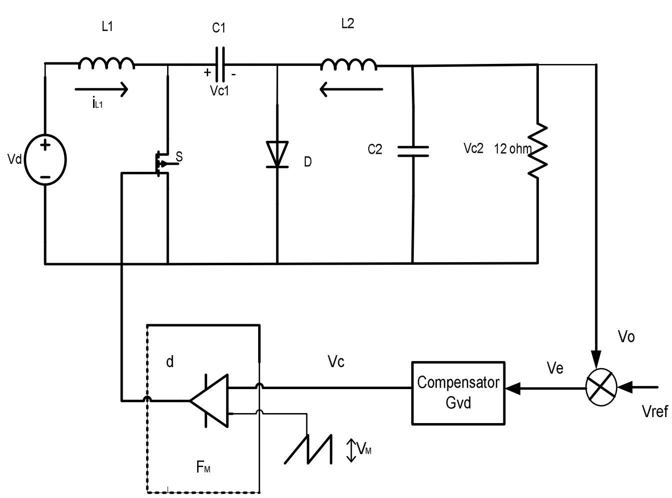

Fig. 2.a Cuk converter with PWM control

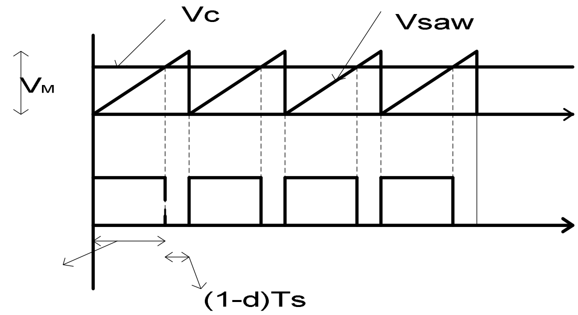

Fig.2.b waveform of PWM comparator

Fig.2

Fig. 3 shows a small-signal block diagram of the converter of Fig. 2(a). The power stage transfer functions are represented by Gvd which is derived earlier. The transfer function of PWM comparator is given by-

$$F_M = \frac{1}{V_M}$$

Where V_M is the amplitude of sawtooth voltage. From Fig 3 the open loop transfer function can be defined as-

$$T(s) = G_c(s) G_{vd}(s) F_M$$

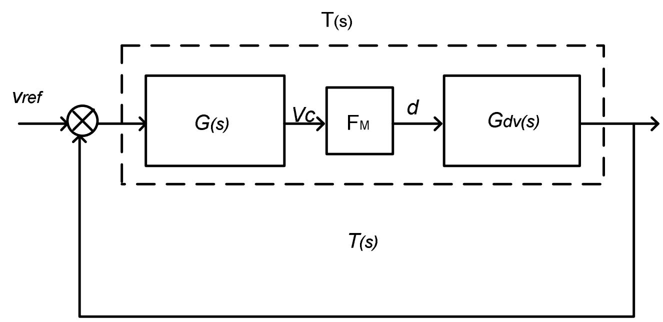

Fig.3

The loop gain *T(s)* is defined as the product of the small signal gains in the forward and feedback paths of the feedback loop. It is found that the transfer function from a disturbance to the output is multiplied by the factor 1/(1+*T(s)*). So the loop gain magnitude $\| T \|$ is a measure of how well the feedback system works.

B. Example[11]

Parameters of the Cuk converter:

Input Voltage Vin	12 Volts
Output Voltage	24 Volts
Switching frequency	100 kHz
Load	12 Ohm
PWM Gain	1/5
L1	68.7 µH
L2	2.2mH
C1	3.7µF
C2	984µF
Output ripple	5%

The transfer function of the converter is obtained from (11) is as follows:

$G_{vd} =$

$$\frac{-814.8s^3 + 2.456\times10^7 s^2 - 1.232\times10^{12} s + 2.154\times10^{16}}{s^4 + 149.4s^3 + 4.922\times10^8 s^2 + 6.25\times10^{10} s + 2.02\times10^{14}}$$

This is the fourth order transfer function. It has two pair of complex pole and three zero in the RHP.

Zeros and poles of the converter are as given as:

Poles

-11.1685 +22175.8678i

-11.1685 - 22175.8678i

-63.5356 +637.7841i

-63.5356 - 637.7841i

Zeros are-

4995.0229 +35864.4541i

4995.0229 - 35864.4541i

20156.5294

C. Model Reduction:

Using Pade-Approximation method [13], the reduced order transfer function of the converter is obtained as follows:

$$G_{1vd} = \frac{-2511s + 4.378\times10^7}{s^2 + 127s + 410600} \qquad (15)$$

Poles and zeros of reduced system-

Poles-

-63.5 + 637.6266i

-63.5 - 637.6266i

Zero

17435.2847

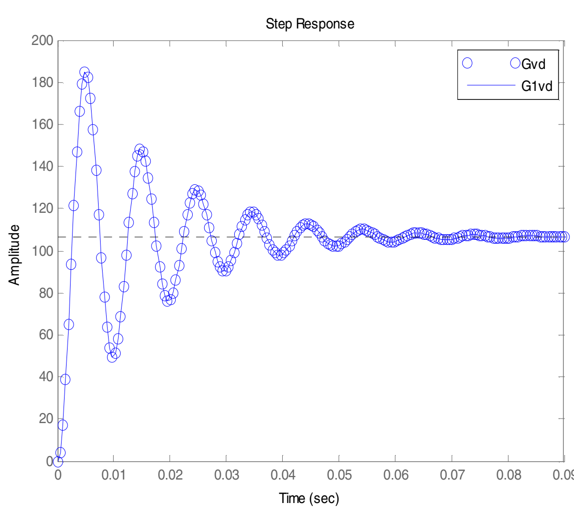

Fig. 4 Step response of reduced and full order system

Integral Square Error (ISE) between original system and reduced order system is 0.003249

D. Feedback controller design

In this paper voltage-mode linear averaged feedback controllers for dc–dc converter is designed in frequency domain. The main objective of the controller design is to obtain stable operation of the converter by varying the duty cycle. Following points are taken care while designing of the compensator.

(1) The averaged mathematical model is accurate up to one tenth of switching frequency. Here the switching frequency is taken as 100 kHz therefore the bandwidth (0 dB cross over frequency of closed loop system) should be near 10kHz.

(2) High gain at low frequency region provides good output voltage regulation. And phase margin determines the transient response to sudden change in input voltage. The suitable phase margin is between 45^0 to 60^0 degree.

Fig.5 shows Bode plot without compensator that has both gain margin and phase margin negative. To make the gain margin and phase margin positive suitable poles and zeros of compensator is selected.

Fig.5 shows complex pair of pole that occurs at 640 rad/sec which provide 180^0 phase lag. To overcome, this problem two zeros is added at 640 rad/sec in compensator. To minimize the effect of noise at high frequency one pole is added at high frequency. To provide good output regulation one pole is added at very low frequency.

With these considerations the designed compensator is

$$G_c = \frac{400(\frac{s}{640}+1)^2}{s(\frac{s}{2\times10^5}+1)}$$

and therefore, the overall open-loop transfer is

$$T(s) = \frac{-0.4904s^3 + 7923s^2 + 1.074\times10^7 s + 3.502\times10^9}{5\times10^{-6}s^4 + 1.001s^3 + 129.1s^2 + 410600s}$$

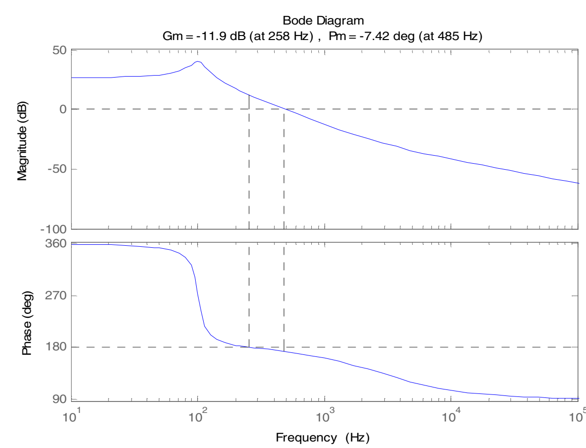

Fig.5

Figure 5 shows the Bode plot of uncompensated open loop system which has gain margin -11.9dB and phase margin -7.42 deg.

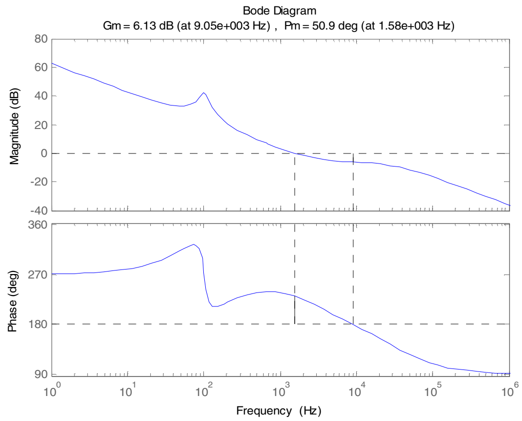

Fig.6

Figure (6) shows Bode diagram of compensated open loop system.

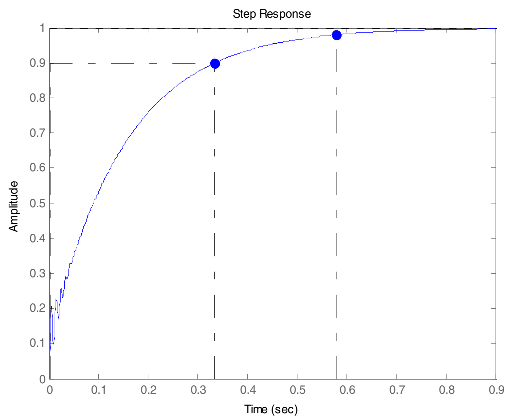

Fig.7 Step response of original system with compensator.

Conclusion

This paper deals with modeling and control of Cuk converter operating in continuous conduction mode (CCM). The state space averaging technique is applied to find out the linear model of Cuk converter and the desired transfer function in terms of duty ratio to output voltage (G_{vd}) is obtained which is a fourth order transfer function. Designing a compensator for the fourth order system is very difficult. Therefore, fourth order transfer function of Cuk converter is reduced to second order and it is found that step response of reduced order model closely follow the original system. The compensator designed for second order system gives quite satisfactory response with the original system.

REFERENCES

[1] D. W. Hart, *Introduction to Power Electronics*, Prentice Hall, Inc., 1997.

[2] R.W. Erickson and D. Maksimovic, *Fundamentals of Power Electronics,* 2nd ed., Kluwer Academic Publishers, 2001.

[3] R. D. Middlebook and S. Cuk, "A General Unified Approach to Modeling Switching-Converter Power Stages" *International Journal of Electronics*, vol. 42, pp. 521-550, June 1977.

[4] V. Vorpérian, "Analysis of the Sepic converter by Dr. Vatché Vorpérian," *Ridley Engineering Inc*, www.switchingpowermagazine.com, 2006.

[5] R. Ridley, "Analyzing the sepic converter," *Power Systems Design Europe Magazine*, pp. 14-18, November 2006.

[6] A. Hren and P. Slibar, "Full order dynamic model of sepic converter," *Proc. of the IEEE International Symposium on Industrial Electronics*, pp. 553-558, June 2005.

[7] E. Vuthchhay, P. Unnat, and C. Bunlaksananusorn, "Modeling of a sepic converter operating in continuous conduction mode," *6th International Conference on Electrical Engineering/Electronics, Computer, Telecommunications and Information Technology 2009 (ECTI-CON 2009)*, pp. 136-139, May 2009

[8] V. Vorperian, "Simplified analysis of PWM converters using model of PWM switch, Part I and Part II: Discontinuous conduction mode," *IEEE trans. on Aerosp. Electron. Syst.*, July 1990.

[9] E. Vuthchhay, C. Bunlaksananusorn, and H. Hirata "Dynamic Modeling and Control of a Zeta Converter," *International Symposium on Communications and Information Technologies 2008 (ISCIT 2008)*, Oct. 2008.

[10] A. J. Forsyth and S.V. Mollov, "Modelling and control of DC-DC converters," *IEEE Power Engineering Journal*, pp. 229-236, 1998.

[11] B. C. Kuo, *Automatic Control Systems*, 7th ed., Prentice Hall Inc, 1995.

[12] A. Chuinard, T. Chairet, "Feedback compensation techniques to improve input disturbance response in the Cuk converter," June . 2009

[13] Shamash, Y, "Stable reduced-order models using Padé- type approximations," *Automatic Control, IEEE Transactions*, vol.19, no.5, pp. 615- 616, Oct 1974.