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Benefit reentitlement conditions in unemployment insurance schemes*

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Abstract

The past employment history - employment requirements - is part of the eligibility conditions for unemployment insurance in most western countries. In a standard search-matching model, we show how employment requirements strengthen the reentitlement effect and thereby changes the trade-off between insurance and incentives in the design of the optimal insurance scheme. Deploying employment requirements for benefit eligibility may thus allow for both higher benefit levels and longer duration, and yet labor market performance is improved. When the need for insurance increases due to higher risk aversion, employment requirements becomes less lenient, and oppositely when the environment becomes more risky.

JEL: E32, H3, J65

Keywords: Reentitlement effects, incentives, job-search, unemployment insurance.

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1 Introduction

In the literature, unemployment benefit schemes are usually characterized in terms of benefit levels and duration. In most unemployment benefit systems throughout the western world eligibility conditions are a crucial design feature. Systems often have entry conditions specified in terms of education and/or employment as well as employment requirements (e.g. a certain number of hours of work within a preceding reference period) to initiate a new benefit spell, e.g. (see e.g. Venn (2012) for details). The duration of benefits has to be seen relative to the employment requirement. If this requirement is lax, the maximum duration of a benefit spell matters less since it is easy to regain eligibility and vice versa. This suggests that employment requirements are an important aspect of the unemployment insurance system affecting the incentive structure and hence labour market outcomes. Employment requirements have also gained policy interest with a particular focus on the possibility that tough employment requirements may reduce the incentive to accept short-term jobs (Danish Commission on Unemployment Insurance (2015)).

Since the seminal paper by Mortensen (1977) introducing the concept of re-entitlement in a model with finite benefit duration, it has been studied how the entitlement effect affects the optimal design of the unemployment insurance scheme (see e.g. Fredriksson & Holmlund (2001) and Coles & Masters (2006)). While the focus of these and related studies has been on the benefit profile, less attention has been devoted to the effects of the design of the employment requirements. In a set of papers, Hopenhayn & Nicolini (1997, 2009) addressed the issue of employment requirements in an optimal unemployment insurance system by introducing a wage tax on earnings that are dependent on previous unemployment. The optimal unemployment insurance system is shown to have benefits that decreases and a wage tax that increases in the length of the unemployment spell. In Hopenhayn & Nicolini (2009) it is shown that difficulties in distinguishing between quits and layoffs make it optimal to condition benefit eligibility on the employment history of unemployed.

This paper is related to Hopenhayn & Nicolini (1997, 2009), but focuses on a more restricted set of unemployment insurance contracts that are closer to what is observed in unemployment insurance systems around the world. That is, we do not consider the possibility of imposing an unemployment dependent wage tax, but instead investigate how an employment requirement for re-entitlement affects the optimal design of unemployment system and how this relates to other parts of the unemployment insurance system. Our paper is closer to Ortega & Rioux (2010). They emphasize that an employment criterion can support job creation since unemployed who have exhausted their benefits are willing to accept lower wages to regain the right to unemployment benefits and they show how an employment criterion is part of the optimal unemployment insurance system. They, however, ignore the search effects of changing the parameters of the benefit system. Since the effects of unemployment insurance on job search incentives is crucial we allow for endogenous search.

We thus ask the basic question how employment requirements as part of the unemployment insurance system affect the incentive structure. A finite duration of benefits produces the standard entitlement effect which strengthens job-search incentives for unemployed. The presence of an employment requirement strengthens this effect for those meeting the requirement. The importance of finding a job becomes larger, if it ensures that the employment requirement for eligibility remains fullfilled. If a job is not found, not only will benefits be lower in the remaining unemployment spell, but benefits will also be low in subequent unemployment spells, since a new job spell does not automatically ensure that the employment criterion is met. Oppositely, search incentives weakens for the unemployed who do not automatically meet the employment crietion if finding a job. We explore these effects and show that including employment requirements have a significant effect on how to balance incentives and insurance in the design of the optimal unemployment insurance scheme, and therefore also on labour market outcomes.

We analyse the role of employment requirements as part of the eligibility criterion for unemployment benefits in an otherwise standard search-matching framework (see e.g. Mortensen (1977) and Fredriksson & Holmlund (2001)). This framework is well-known and adopting it allows us to focus on the role of employment requirements compared to those of benefit levels and duration. Note that in the standard search-matching model it is (implicitly) assumed that any work history qualifies for a new benefit period. It is clear, that for employment eligibility requirements to have importance, the benefit scheme has to include at least two-tiers; the benefit level (unemployment benefits) for those meeting the requirement, and a lower level (social assistance) for those who do not. Similarly, a finite benefit duration implies a possible transit from the higher to the lower benefit level (a simple form of time dependent benefits). The main contribution of our paper is to compare the incentive effects of employment requirements to those of benefit levels and duration. We show that the employment requirement can work as a substitute to the benefit duration in terms of generating incentives for job search and thus affect the trade-offs underlying the optimal design of the unemployment insurance system. The optimal scheme is shown to be markedly different when the employment requirement is an active instrument in the design of the unemployment insurance system. We consider the design of the unemployment insurance scheme and how it depends on the need for insurance measured by risk aversion or job separations in the labour market. The former captures preferences and the latter the environment and the possible effects on unemployment insurance scheme from more volatility in labour markets. It is shown that although both changes call for more insurance, the adoption of the optimal unemployment insurance scheme differs between the two cases.

The structure of the paper is as follows: In Section 2 we introduce employment conditions in a search-matching model. The effects of such requirements are analyzed in Section 3 and based on a calibration for Denmark we analyse in Section 4 the optimal unemployment insurance scheme, and compare the outcome to the case where there is no such conditions. Concluding remarks are given in Section 5.

2 Benefit entitlement in a search-matching model

Consider a search-matching model of the labor market where workers can be in one of four states: *i*) possessing a job and fulfilling the employment requirement for benefit eligibility (b_U) in case of involuntary job separation (state E), *ii*) possessing a job without fulfilling the employment requirement and thus being entitled to social benefits ($b_K < b_U$) rather than unemployment benefits ($b_U > b_K$) in case of involuntary job separation (state N), *iii*) being unemployed and entitled to unemployment benefits, UIB-unemployed (state U), and *iv*) being unemployed and not entitled to unemployment benefits but social assistance, SA-unemployed (state K).[1] There is a continuum of workers with mass one. All workers are assumed to own an equal share of the firms, and therefore firm profits are distributed among workers, allowing a general equilibrium analysis. All employed may lose their

[1] For simplicity we do not consider a state where an unemployed receiving benefits (b_U) transits to jobs of type N. In many countries entitlement requires a given amount of employment within a certain period to qualify for benefits and it is therefore possible that the subsequent employment period is too short to requalify for the high benefits. This simplification may affect the implications of the employment criterion, but are not expected to alter the main conclusions of the analysis.

job by the exogenous separation rate[2] $p_{U,E}$. Employed not entitled to unemployment benefits gain eligibility[3] at the rate $p_{E,N}$, and thus the expected length of the employment period to regain UIB eligibility is $1/p_{E,N}$. Unemployed eligible for benefits search for jobs at the rate s_U and find a job at the rate αs_U, where α is the job-finding rate (see below). Unemployed eligible for benefits lose eligibility at the rate[4] $p_{K,U}$, and thus the expected potential duration of benefit receipt is $1/p_{K,U}$. Finally, unemployed non-eligible for benefits search at the rate s_K and thus find a job at the rate αs_K. Note that there are only involuntary job separations in the model.

The instantaneous utility to an employed is

$$h(I_i, 1-l) \ \ ; \ \ i = E, N$$

where I_i is net income given as labor income after tax[5] $w_i [1-\tau]$ plus its share of profits Π. Time endowment is normalized to unity and working hours are l (exogenous).

The instantaneous utility for unemployed is[6]

$$g(I_j, 1-s_j) \ \ ; \ \ j = U, K$$

where income is the sum of the transfer (b_U or b_K, $b_U > b_K$) plus the profit share Π, and s_j is the amount of time spent searching for a job. The functions h and g have standard properties.

[2]This assumption is made to avoid introducing other differences between workers e.g. based on temporary vs. permanent contracts, to focus on the effect of the design of the unemployment insurance scheme.

[3]This formulation avoids having to make eligibility history dependent which in turns introduces heterogeneity across otherwise identical individuals, see Andersen and Ellermann-Aarslev (2018). A stationary environment is considered. Allowing for entry (young) and exit (old) to the labour market would imply that there is an inflow of young not meeting the employment requirement and outflow of old who do. Explicitly including this would complicate the analysis considerably.

[4]That is, we follow Fredriksson & Holmlund (2001) who show that a fixed time duration can be approximated by a system in which there is a stochastic transition from one benefit level to another.

[5]Labour income is taxed while profits is not. Note that the tax is financing expenditures on unemployment benefits and social assistance. Hence, this formulation captures actual modes of financing via social contributions or taxes.

[6]For notational reasons, we allow the instantaneous utility function of the unemployed to differ from that of the employed. However, our results will not hinge on this asymmetry, and in the numerical illustrations we assume $h(\cdot) = g(\cdot)$.

The value functions associated with the four possible labor market states are

$$\begin{aligned}
\rho V_E &= h\left(w_E\left[1-\tau\right]+\Pi, 1-l\right)+p_{U,E}\left[V_U-V_E\right] \\
\rho V_N &= h\left(w_N\left[1-\tau\right]+\Pi, 1-l\right)+p_{U,E}\left[V_K-V_N\right]+p_{E,N}\left[V_E-V_N\right] \\
\rho V_U &= g\left(b_U+\Pi, 1-s_U\right)+\alpha s_U\left[V_E-V_U\right]+p_{K,U}\left[V_K-V_U\right] \\
\rho V_K &= g\left(b_K+\Pi, 1-s_K\right)+\alpha s_K\left[V_N-V_K\right]
\end{aligned}$$

where ρ is the discount rate.[7]

To focus on differences or asymmetries arising solely from the design of the unemployment insurance system, we assume that all workers are identical except for their labor market history and thus possibly their benefit entitlement. In this spirit we also assume that $p_{K,N} = p_{U,E}$; i.e., the job separation rate is the same for eligible and non-eligible workers, and that they have the same working hours (exogenous).[8]

In the following this short-hand notation will be used

$$\begin{aligned}
h_i(\cdot) &\equiv h\left(w_i\left[1-\tau\right]+\Pi, 1-l\right) \text{ for } i=E,N \\
g_j(\cdot) &\equiv g\left(b_j+\Pi, 1-s_j\right) \text{ for } j=U,K.
\end{aligned}$$

The individual takes all variables except the search level to be beyond its own control (i.e., to be unaffected by its decisions), and thus the optimal search level for the two types of unemployed is determined by (note that standard assumptions on g ensure that the second order condition is fulfilled)

$$\frac{\partial g_U(\cdot)}{\partial(1-s_U)} = \alpha\left[V_E-V_U\right] \quad (1)$$

$$\frac{\partial g_K(\cdot)}{\partial(1-s_K)} = \alpha\left[V_N-V_K\right]. \quad (2)$$

Denoting the share of the population receiving unemployment benefits and social assistance by u and k, respectively, we have that total search is given as

$$s \equiv s_U u + s_K k.$$

A standard constant returns to scale matching function defined over total search and vacancies (v) is assumed

$$m(s,v).$$

[7]Note also that the participation constraints: $V_E \geq V_U, V_N \geq V_K, V_E \geq V_N, V_U \geq V_K$ are assumed fulfilled in the following. This is checked in the calibrations reported below.

[8]We impose this symmetry to focus on the question whether re-entitlement conditions can be motivated as a means to improve the trade-off between incentives and insurance in the social safety net.

The function m has the usual properties. It follows that the job-finding rate is given by

$$\alpha = \frac{m(s,v)}{s} = m(1,\theta)$$

where $\theta \equiv \frac{v}{s}$ is market tightness, and hence $\alpha = \alpha(\theta)$, $\alpha'(\theta) > 0$. The job filling rate is

$$q = \frac{m(s,v)}{v} = m\left(\theta^{-1}, 1\right)$$

and thus $q = q(\theta)$, $q'(\theta) < 0$.

Firms post vacancies to find vacant workers, and in the hiring process they are not able to distinguish workers by their eligibility in the unemployment insurance scheme. Thus, the value of a vacancy (J_V) is expressed in terms of the expected value of a filled job (J_{EXP}); i.e.,

$$\rho J_V = -\kappa + q\left[J_{EXP} - J_V\right]$$

where κ is the flow vacancy cost. The free-entry-condition, $J_V = 0$, then implies

$$J_{EXP} = \frac{\kappa}{q} \tag{3}$$

where J_{EXP} is the expected value of a filled job; that is

$$J_{EXP} \equiv \frac{\alpha s_U u J_E + \alpha s_K k J_N}{\alpha s_U u + \alpha s_K k} = \frac{s_U u J_E + s_K k J_N}{s_U u + s_K k}.$$

After a firm and a worker are matched, the firm knows whether the candidate is entitled to UIB or SA, and therefore the wage depends on the worker's UIB eligibility. The value of a job filled with an UIB eligible worker is[9]

$$\rho J_E = y - w_E + p_{U,E}\left[J_V - J_E\right],$$

and the value of a job filled with a worker non-eligible for unemployment benefits is

$$\rho J_N = y - w_N + p_{E,N}\left[J_E - J_N\right] + p_{U,E}\left[J_V - J_N\right].$$

Wages are determined through Nash bargaining; i.e.,

$$\begin{aligned} w_E &= \arg\max_{w_E} \left(V_E - V_U\right)^{\beta} \left(J_E - J_V\right)^{1-\beta} \\ w_N &= \arg\max_{w_N} \left(V_N - V_K\right)^{\beta} \left(J_N - J_V\right)^{1-\beta} \end{aligned}$$

[9]Note that workers gaining eligibility for b_U experience an immediate change in the wage since the worker and the firm are implicitly assumed to renegotiate the wage promptly. This assumption may be empirically questionable, but we make it for tractability reasons.

with the associated first-order conditions (second order conditions assumed to be fulfilled)

$$\beta\frac{\frac{\partial V_E}{\partial w_E}}{V_E - V_U} + (1-\beta)\frac{\frac{\partial J_E}{\partial w_E}}{J_E} = 0 \tag{4}$$

$$\beta\frac{\frac{\partial V_N}{\partial w_N}}{V_N - V_K} + (1-\beta)\frac{\frac{\partial J_N}{\partial w_N}}{J_N} = 0. \tag{5}$$

Profits are given as

$$\Pi = [y - w_E]\, e + [y - w_N]\, n - v\kappa$$

where e and n denote the number of eligible and non-eligible workers, respectively. The inflow and outflow equations read (where $e = 1 - u - k - n$)

$$U \;:\; ep_{U,E} = \alpha s_U u + p_{K,U} u \tag{6}$$

$$K \;:\; np_{U,E} + p_{K,U} u = \alpha s_K k \tag{7}$$

$$N \;:\; \alpha s_K k = p_{U,E} n + p_{E,N} n \tag{8}$$

for unemployment, social assistance and non-eligible jobs, respectively.

For later reference note that the fraction of the population in the various labor market states can be written (recall that $1 = e + n + u + k$)

$$\begin{aligned} e &= e(\alpha, s_U, s_K, p_{E,N}, p_{K,U}, p_{U,E}) \\ n &= n(\alpha, s_U, s_K, p_{E,N}, p_{K,U}, p_{U,E}) \\ u &= u(\alpha, s_U, s_K, p_{E,N}, p_{K,U}, p_{U,E}) \\ k &= k(\alpha, s_U, s_K, p_{E,N}, p_{K,U}, p_{U,E}). \end{aligned}$$

Finally, the public budget constraint reads[10]

$$\tau\,(w_E e + w_N n) = b_U u + b_K k. \tag{9}$$

To sum up, the unemployment insurance scheme is characterized by two benefit levels (b_U, b_K), the transition rate out of b_U $(p_{K,U})$ determining benefit duration, and the entry rate into b_U eligibility $(p_{E,N})$ capturing reentitlement requirements. Recall that expected benefit duration is $1/p_{K,U}$ and the expected employment requirement period $1/p_{E,N}$.

[10] A non-linear relation (Laffer curve) may arise in general equilibrium between the tax rate and total revenue leading to multiple equilibria. In the calibrations reported below this has been carefully checked, also to ensure that the solution for all variables are economically meaningful (e.g. non-negative).

In summary, the equilibrium to the model is characterized by unemployed choosing search effort according to (1) and (2), firms creating vacancies according to (3), wages determined by (4) and (5), the tax rate determined from (9) and the flow equations (6), (7), and (8). It can be shown[11] that the resource balance condition (or goods-market equilibrium condition) is fulfilled; i.e., aggregate output (net of vacancy costs) equals aggregate consumption.

2.1 Calibration

We calibrate the model to Danish data. The time period is one month. As is standard in the literature, productivity (y) is normalised to 1. Following Shimer (2005), the discount rate (ρ) is set to 0.004 corresponding to an annual rate of 5 %. The flow cost of vacancies (κ) is set to the value estimated by Ortega & Rioux (2010), i.e. 37 per cent of output. For the job separation rate ($p_{U,E}$) we use the estimate from Rosholm & Svarer (2004), i.e. 0.008. We assume the following functional forms for instantaneous utility

$$\begin{aligned} h\left(I_i, 1-l\right) &= \frac{1}{1-\gamma} I_i^{1-\gamma} + \log\left(1-l\right) \quad ; \quad i = E, N \\ g\left(I_i, 1-s_i\right) &= \frac{1}{1-\gamma} I_i^{1-\gamma} + \log\left(1-s_i\right) \quad ; \quad i = U, K \end{aligned}$$

In the benchmark model, we set $\gamma = 1.5$, but below we consider a range of different γ's. Workers are assumed to spend 40 per cent of their time at work, i.e. $l = 0.4$.

The matching function is of the standard Cobb-Douglas form

$$m\left(s, v\right) = A s^{\varepsilon} v^{1-\varepsilon}.$$

We set $\beta = \varepsilon = 0.5$ imposing a Hosios (1990)-type condition.[12] In the benchmark, the four instruments of the UI system are set to match the Danish UI system. The standard duration of UI benefits is two years, and the work requirement to regain entitlement to UI is one year, i.e. we set $p_{K,U} = 0.04167$ and $p_{E,N} = 0.08333$. The UI level (b_U) and the SA level (b_A) are set according to the evidence from the Danish Commission on Unemployment Insurance (2015), i.e. we aim for after-tax replacement rates of 62.8 per cent for workers entitled to UI benefits and 35.4 for workers only entitled to SA. Finally, we calibrate the matching efficiency parameter (A) to obtain a gross unemployment rate

[11] For proof see Appendix available upon request from the authors.

[12] Note that this condition does not guarantee efficiency in the case of risk averse agents and heterogeneous workers in the same market.

$(u + k)$ of 6.2 per cent, which is the average gross unemployment rate in Denmark over the period 2007 to end-2016 according to Statistics Denmark. The calibration results in $b_U = 0.551$, $b_K = 0.254$ and $A = 0.103$. The model is non-linear which in general may give rise to multiple equilibria. We have carefully checked[13] that the model only has the unique equilibrium which is reported.

3 Search and properties of the UIB system

The search-matching general equilibrium model admits few analytical results, and we follow the literature in calibrating the model. However, some partial equilibrium results can be found, and since they provide intuition on the effects of employment requirements as part of the unemployment insurance scheme we first report them.

Consider the role of benefit duration $(p_{K,U})$ and benefit entitlement $(p_{E,N})$ for given benefit levels (b_U, b_K) for labor market performance; that is, we consider the implication for a given macroeconomic environment, i.e., wages (w_E, w_N), taxes (τ), and job-finding rate (α). This clarifies the direct incentive/search effects of the two instruments as part of the social safety net.

It can be shown[14] that for a given macroeconomic environment

$$\frac{ds_U}{dp_{K,U}} > 0 \ ; \ \frac{ds_K}{dp_{K,U}} < 0.$$
$$\frac{ds_U}{dp_{E,N}} < 0 \ ; \ \frac{ds_K}{dp_{E,N}} > 0.$$

First, this confirms the standard effect that shorter benefit duration (higher $p_{K,U}$) makes the UIB-unemployed search more to enhance the chance of finding a job in light of the more dire consequences if this is not successful. For the SA-unemployed, search becomes less attractive since the value of finding a job leading to UIB entitlement is now lower. Second, and new, a more lax employment requirement (higher $p_{E,N}$) makes the SA-unemployed search more since the value of a job is now higher since it more easily leads to UIB entitlement. Oppositely the UIB-unemployed search less since it is less critical to lose

[13] We use the routine SolveNLE in the software Ox to solve the system. For reasonable starting values, the solution quickly converges to the one reported in the paper. For other starting values, the routine either does not converge to a solution or it converges to an economically irrelevant solution (e.g. negative search levels). We have also checked the solution using other algorithms and other software. As an example, the frequently used fsolve routine in Matlab delivers the exact same solution.

[14] For proof of results in this section see Appendix available upon request from the authors.

entitlement as it can more easily be regained. This gives the "impact" effect of changes in employment requirements, and indicates why this dimension of the system may play an important role.

Considering the marginal rate of substitution between the two instruments leaving the utility gain from finding a job unchanged for entitled and non-entitled, respectively, we have (for proof see Appendix)

$$\frac{dp_{K,U}}{dp_{E,N}} \mid_{[V_E - V_U]=\text{constant}} > 0.$$
$$\frac{dp_{E,N}}{dp_{K,U}} \mid_{[V_N - V_K]=\text{constant}} > 0.$$
$$\frac{dp_{K,U}}{dp_{E,N}} \mid_{[V_E - V_U]=\text{constant}} \neq \frac{dp_{E,N}}{dp_{K,U}} \mid_{[V_N - V_K]=\text{constant}} .$$

Note that search levels are unchanged for utility gains being constant ($V_E - V_U$ = constant, and $V_N - V_K$ = constant). The above thus gives the combinations of $p_{K,U}$ and $p_{E,N}$, leaving search (s_U and s_K respectively) unchanged. Both types' iso-search curves are upward sloping in the $(p_{K,U}, p_{E,N})$-space; that is, a higher rate ($p_{E,N}$) at which non-entitled become entitled to UIB (easier UIB entitlement) has to be accompanied by a higher rate ($p_{N,U}$) at which UIB-unemployed lose their UIB entitlement (shorter benefit duration) to leave search unchanged, i.e., the two instruments are substitutes. This applies for both types of unemployed, but the marginal rates of substitution are different, which suggests that it may be desirable to let the UI scheme feature both elements (see below). Intuitively, using two instruments dominates using only one instrument since there are two search levels (they are in general different) which can be affected, and the two instruments are not perfect substitutes, cf. the difference in the marginal rates of substitution.

Finally, note that (for proof see Appendix) shorter benefit duration (higher $p_{K,U}$) makes both the eligible employed (V_E decreases) and non-eligible unemployed (V_K decreases) worse off, the former because unemployment causes a larger drop in utility and the latter because the gain from finding a job decreases. The effect on V_N and V_U is ambiguous. Oppositely, a more lax employment requirement (higher $p_{E,N}$) leads to an increase in V_E and V_K, while there is an ambiguous effect on V_N and V_U. This suggests that the two instruments have different distributional consequences.

3.1 Labor market outcomes

Consider next the role of benefit duration and employment eligibility conditions for labour market performance assessed in terms of the gross level of unemployment $(u + k)$, i.e. recipients of unemployment benefits or social assistance.

From the flow equations we have

$$[e + n]\, p_{U,E} = \alpha s_U u + \alpha s_K k = \alpha s$$

i.e., for a given job separation rate $p_{U,E}$ and job-finding rate α, total employment $(e+n)$ is monotonously increasing in aggregate search. Since $1 = e + n + u + k$ it follows that $u + k$ is monotonously decreasing in s.

We show in the Appendix that the gross unemployment rate $(u+k = 1-(e+n))$ can be written in implicit form as

$$u + k = F(s_U\left(p_{K,U}, p_{E,N}\right), s_K\left(p_{K,U}, p_{E,N}\right), p_{K,U}, p_{E,N}).$$

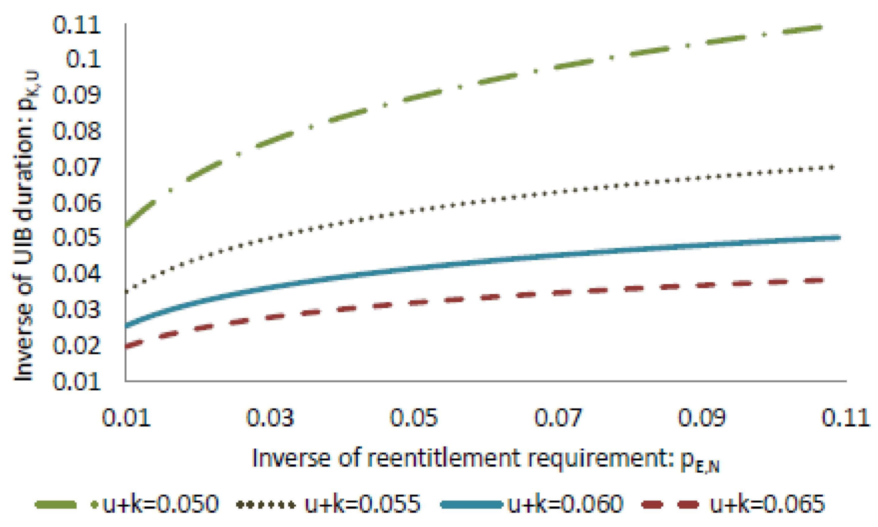
where

$$\begin{aligned} \frac{\partial F(\cdot)}{\partial s_U} &< 0 \;;\; \frac{\partial F(\cdot)}{\partial s_K} < 0. \\ sign\left(\frac{\partial F(\cdot)}{\partial p_{K,U}}\right) &= sign(s_U - s_K). \\ sign\left(\frac{\partial F(\cdot)}{\partial p_{E,N}}\right) &= -sign\left(\frac{\partial F(\cdot)}{\partial p_{K,U}}\right). \end{aligned}$$

Increases in search by either type unambiguously reduces gross unemployment. Whether shorter benefit duration (higher $p_{K,U}$) reduces gross unemployment depends on the difference in search levels between recipients of unemployment benefits and social assistance. If the UIB-entitled search less than the SA-entitled ($s_U - s_K < 0$; as is the case in the calibration reported below), shorter benefit duration decreases gross unemployment. The effect on gross unemployment of changes in benefit duration ($p_{K,U}$) and entitlement conditions ($p_{E,N}$) are always oppositely signed since the two are substitutes, cf. above. Hence, if the UIB-entitled search less than the SA-entitled, more lax entitlement conditions (higher $p_{E,N}$) increase gross unemployment.

The finding that benefit duration and eligibility conditions are substitutes also holds in general equilibrium (endogenous search, wages, taxes etc.), cf. Figure 1 giving iso-gross unemployment curves in the ($p_{E,N}$, $p_{K,U}$)-plane (see above on the calibration). We see that the iso-gross unemployment curves are positively sloped, displaying a trade-off between easier reentitlement and shorter UIB duration in sustaining a certain level of gross unemployment $(u + k)$. The main explanation is that SA-unemployed search more intensively than UIB-unemployed $(s_K > s_U)$, and therefore the response for this group is quantitatively most important. It is thus an implication that the composition of gross unemployment changes along the iso-gross unemployment curves. Along the iso-curves with lower benefit duration (higher $p_{K,U}$) and the more lax the entitlement condition (higher $p_{E,N}$), the lower the number of UIB-unemployed and the larger the number of SA-unemployed. In net terms the effect of shorter benefit duration (exit from the pool of UIB-unemployed) dominates the more lax entitlement condition (entry into the pool of UIB-unemployed). Finally, note that gross unemployment increases when we move to the South-East, i.e., longer UIB duration and/or more lax employment requirements. The trade-off between the two instruments becomes more flat, the higher the gross unemployment rate.

FIGURE 1: ISO-GROSS UNEMPLOYMENT CURVES, TRADE-OFF BETWEEN UIB DURATION AND REENTITLEMENT REQUIREMENT



4 Optimal social safety net

Consider next the optimal design of the social safety net. Assume a utilitarian social welfare function (Ω) given as the sum of utility generated in the economy under a given policy package $(b_U, b_K, p_{K,U}, p_{E,N})$, which can be written[15]

$$\begin{aligned}\Omega &= e\rho V_E + n\rho V_N + u\rho V_U + k\rho V_K \\ &= eh_E(\cdot) + nh_N(\cdot) + ug_U(\cdot) + kg_K(\cdot). \end{aligned} \tag{10}$$

As a prelude it is useful to note that the model includes some important special cases. The standard case considered in the literature assumes that employment automatically gives entitlement to unemployment benefits in the case of layoffs corresponding to $p_{E,N} \rightarrow \infty$, in which case $n \rightarrow 0$. A simple one-tier benefit scheme arises in this case if $p_{K,U} \rightarrow 0$ (infinite benefit duration) implying $k \rightarrow 0$. Note also that $p_{E,N} \rightarrow 0$ implies that it is not possible to transit to a job providing entitlement to unemployment benefits in the case of lay-off, and hence[16] $u \rightarrow 0$ and $e \rightarrow 0$; i.e., this case corresponds to a one-tier benefit scheme where the only two states are N and K. We turn to an exploration of the optimal design of the unemployment insurance scheme.[17]

4.1 Optimal UI scheme

The calibration has the optimal UI scheme to include two tiers and non-automatic entitlement for UIB, i.e., $b_U > b_K > 0$, $0 < p_{K,U} < \infty$, and $0 < p_{E,N} < \infty$. In particular, social welfare (10) is maximized for $b_U = 0.659$, $b_K = 0.328$, $p_{K,U} = 0.0760$, and $p_{E,N} = 0.0150$. Gross unemployment $(u + k)$ is approximately 6%, the replacement rate for UIB eligible workers ($RR_U \equiv \frac{b_U}{w_E(1-\tau)}$) is 75%, and 39% for non-eligible workers ($RR_K \equiv \frac{b_K}{w_N(1-\tau)}$).

The interesting question is the role played by the four dimensions of the unemployment insurance scheme $(b_U, b_K, p_{K,U}, p_{E,N})$ in the optimal policy. Underlying this is a question of insurance vs. incentives. The unemployment insurance scheme is there to protect against income losses in case of unemployment, but at the same time it should be designed so as to support job search incentives. Properties of this trade-off can be explored by varying the parameter (γ) capturing the relative risk aversion of individuals and thus the

[15] For proof see Appendix available upon request from the authors.

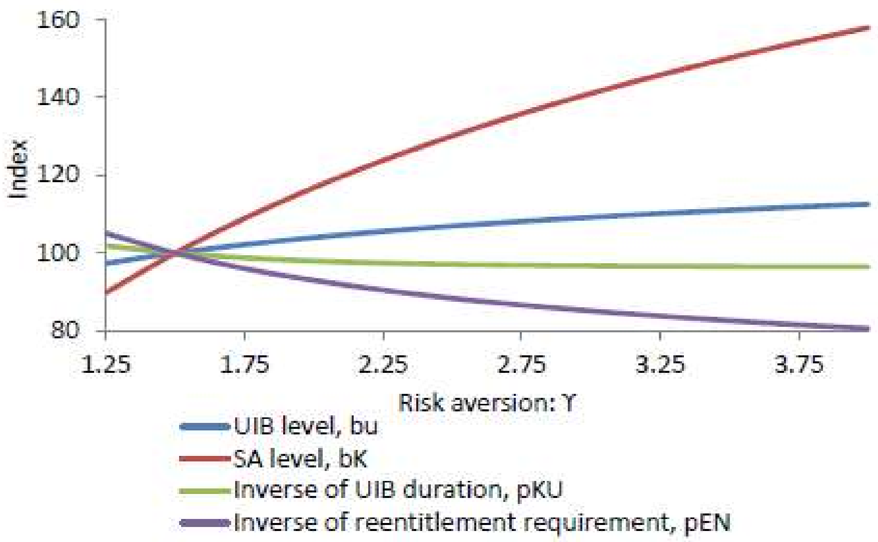
[16] Note that the flow equations imply $np_{E,N} = p_{K,U}u$.

[17] A sufficient condition for the optimal UI scheme to have two tiers is $\lim_{p_{K,U}\rightarrow 0} \frac{\partial\Omega}{\partial p_{K,U}} > 0$. Rigorously speaking, the sufficient condition should also include $\lim_{p_{K,U}\rightarrow\infty} \frac{\partial\Omega}{\partial p_{K,U}} < 0$. However, in the limit there is only notational difference between a one-tier scheme with only SA-unemployed and a one-tier scheme with only UIB-unemployed. A sufficient condition for the optimality of a reentitlement condition is $\lim_{p_{E,N}\rightarrow\infty} \frac{\partial\Omega}{\partial p_{E,N}} < 0$ conditional on two tiers being optimal. There are no readily available analytical results on necessary and sufficient conditions for these conditions to hold.

need for insurance. As is well-known the utilitarian planner redistributes depending on marginal utilities of consumption. By increasing γ, and thus making marginal utilities more sensitive to income/consumption, there is a stronger motive to redistribute. This calls for higher levels of benefits (b_U) and social assistance (b_K). This raises the classical issue of efficiency e. equity. Can and should the disincentive effects of these increases be countered by changes in benefit duration and the employment requirement? Similar questions can be asked if the need for insurance changes due to changes in e.g. job separation rates. We consider both cases.

The response of the four dimensions of the unemployment insurance system to an increase in the risk aversion is shown in Figure 2. While higher risk aversion increases the value of insurance, the optimal policy does not unambiguously become more generous in all dimensions. Higher risk aversion leads to more generous benefit levels (b_U and b_K) as well as longer UIB duration ($p_{K,U}$ decreases), but the eligibility conditions becomes more tight ($p_{E,N}$ decreases). The negative incentive effects of the more generous benefit levels and UIB duration are muted by more tight eligibility conditions.

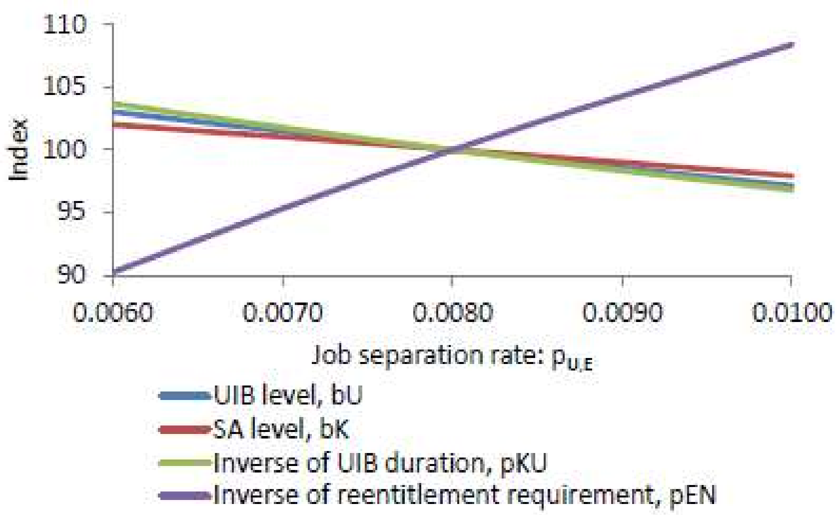
Figure 2: Optimal policy for various degrees of risk aversion (γ)



Note: Variables are indexed relative to the benchmark case, index =100.

A higher job separation rate ($p_{U,E}$) increases the risk of unemployment and thus the value of insurance. Recent policy discussions have focussed much on the need for insurance arising from higher turbulence in labour market due to e.g. globalization and new technologies. The optimal policy changes in this situation to be more generous by having a longer benefit period ($p_{K,U}$ decreases) and more lax entitlement conditions ($p_{E,N}$ increases), whereas it becomes less generous in term of compensation, since both benefit levels (b_U and b_K) decreases, cf. Figure 3.

FIGURE 3: OPTIMAL POLICY FOR VARIOUS LEVELS OF JOB SEPARATION ($p_{U,E}$)



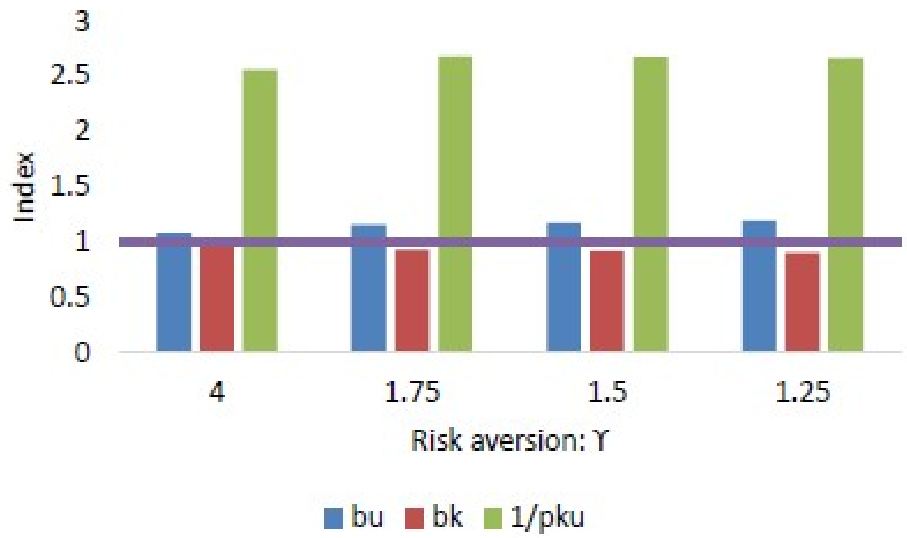
Note: Variables are indexed relative to the benchmark case, index =100.

The different responses of optimal policies to changes in preferences (risk aversion) and the underlying risk (job separation) stresses the interdependencies between the different dimensions of the unemployment insurance scheme. The adjustment of the system to more "demand" for insurance depends critically on the underlying reason (preferences or shocks). Moreover, in either case the system does not move unambiguously in a more generous direction along all dimensions. Incentives matter, and therefore the improved generosity in some dimensions is balanced by tightening in other dimensions. This is also a reminder that simple cross-country comparisons based on a sub-set of dimensions of the entire unemployment insurance scheme and unconditional policy recommendations on the design of the system should be made cautiously.

4.2 The role of employment requirements

Employment conditions are often neglected in analyses of unemployment insurance scheme, implicitly implying that any employment spell is sufficient to gain entitlement for unemployment insurance ($p_{E,N} \to \infty$), cf. the introduction. Is this a useful simplification or does it matter for predictions and policy implications? To address this question we compare the optimal policy in Regime I: where all four dimensions of the scheme are optimized (cf. above) to the case in Regime II where the reemployment condition is removed ($p_{E,N} \to \infty$) and welfare is maximized wrt. the three remaining instruments[18]. This comparison highlights how the active use of the employment requirement affects both the other dimensions of the unemployment insurance scheme but also the labour market outcomes.

FIGURE 4: UNEMPLOYMENT BENEFITS (b_u), SOCIAL ASSISTANCE (b_k) AND BENEFIT DURATION ($1/p_{K,U}$) -REGIME I (EMPLOYMENT REQUIREMENT) VALUES RELATIVE TO REGIME II (NO EMPLOYMENT REQUIREMENT) VALUES



Note: The figure plots the optimal value of the policy instrument in Regime I relative to the optimal value in Regime II.

The inclusion of the employment condition has marked effects on the choice of the other dimensions of the unemployment insurance scheme, cf. Figure 4. Benefit duration ($1/p_{K,U}$) is more than twice as long, unemployment benefits is up to 20% larger, while social assistance is up to 10% lower when the employment requirement is included and

[18]The absence of employment contingencies may release behavioural responses in terms of job-search and quits which are not included.

optimally set. This shows that the trade-off between incentives and insurance changes when including the employment entitlement instrument.

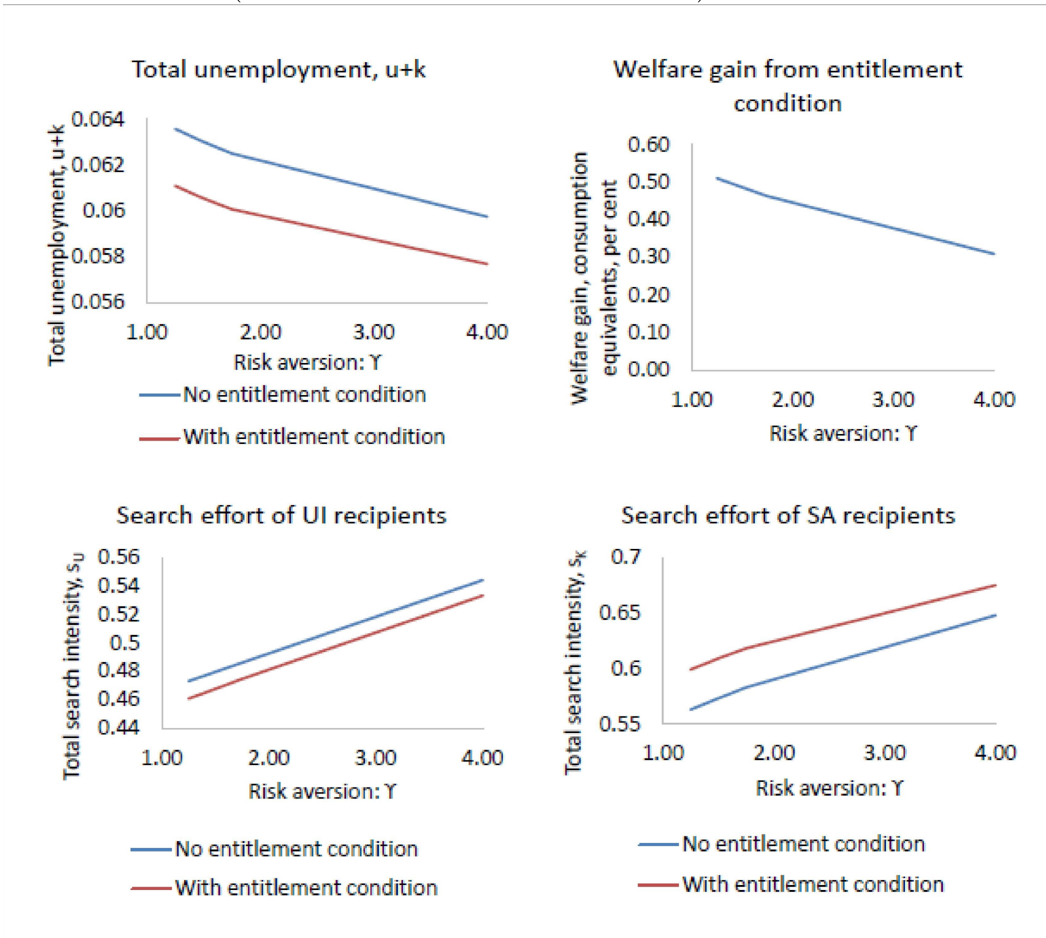
The active use of the employment requirement condition makes those entitled to unemployment insurance better off; higher benefits and longer benefit duration. In contrast, those on social assistance are worse off; lower benefits and it is more difficult to regain the entitlement to unemployment insurance. As a consequence search by UIB-unemployed decreases, while it increases for SA-unemployed. Across the two regimes there is also a wage response. Recipients of unemployment benefits have a better outside opportunity and the search friction implies that they can appropriate some of the improvement in a higher wage, and oppositely for workers without the unemployment benefit entitlement. The latter dominates, and this wage response is conducive for job creation.

All of this implies that gross unemployment $(u + k)$ is reduced as a result of including the optimal employment requirement, cf. Figure 5. It is a noteworthy implication that the composition of gross unemployment changes: in the regime without entitlement condition SA-unemployed constitute about 60% of gross unemployment while with the entitlement condition it falls to 45%. Loosely, UIB-unemployed may be termed short-term and SA-unemployed long-term unemployed, and it is interesting that the design of the unemployment insurance scheme in this way changes the relative size of the two groups. This suggests that entitlement conditions have important distributional effects (which may not be properly captured by an utilitarian social welfare function).

Assessed in terms of ex ante welfare there is a welfare improvement measured in consumption equivalents[19] of about 0.3% ($\gamma = 4$) up to 0.5% ($\gamma = 1.25$). All of this shows that an explicit consideration of the employment criterion has an important effect on the policy trade-offs involved in designing the unemployment insurance system. For comparison note that the welfare loss from not varying either the UI duration or the social assistance benefit level is neglegible (respectively $0,00$ and 0.01 per cent in consumption equivalents for $p_{U,E} = 0.006$).

[19] Seen in relation to the discussion of the welfare costs of business cycle fluctuations (see e.g. De Santis (2007)), this is a rather large effect from just one dimension of the unemployment insurance system.

Figure 5: Outcomes in Regime I (employment requirement) and regime II (no employment requirement) values

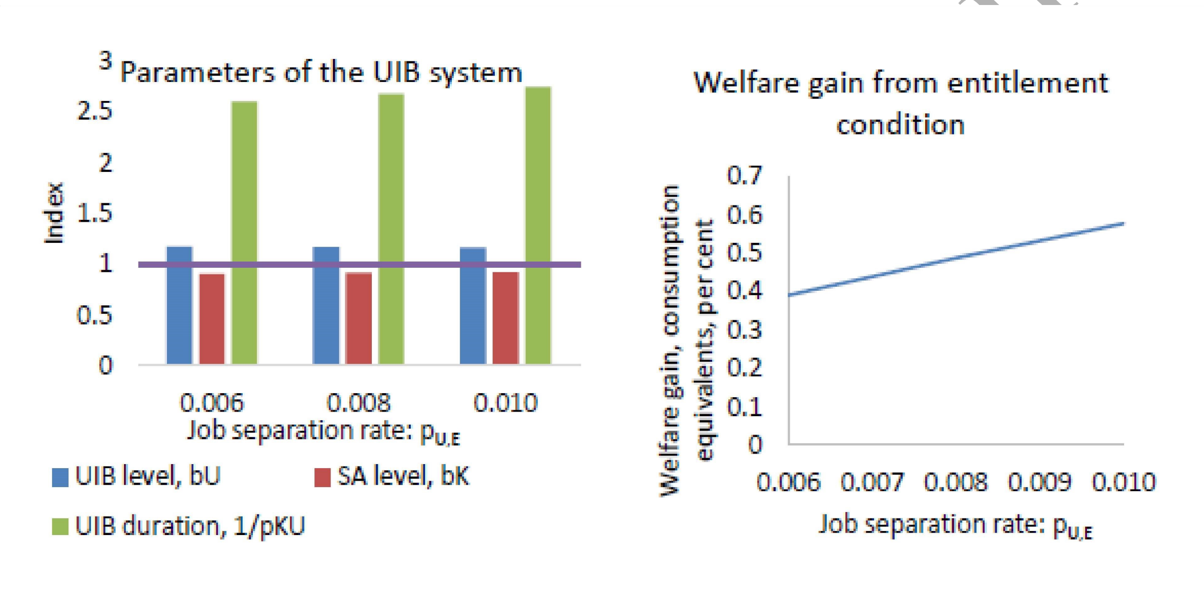


4.3 Robustness analyses

We have carried out a number of robustness checks. A general lesson is that there exists a trade-off between the various UI instruments across parameter sets, but the exact trade-off between incentives and insurance, and thereby the optimal UI policy, depends heavily on the specific calibration. We report here two robustness checks, the role of employment requirements for different values of the job separation rate ($p_{U,E}$) and the discount rate (ρ).

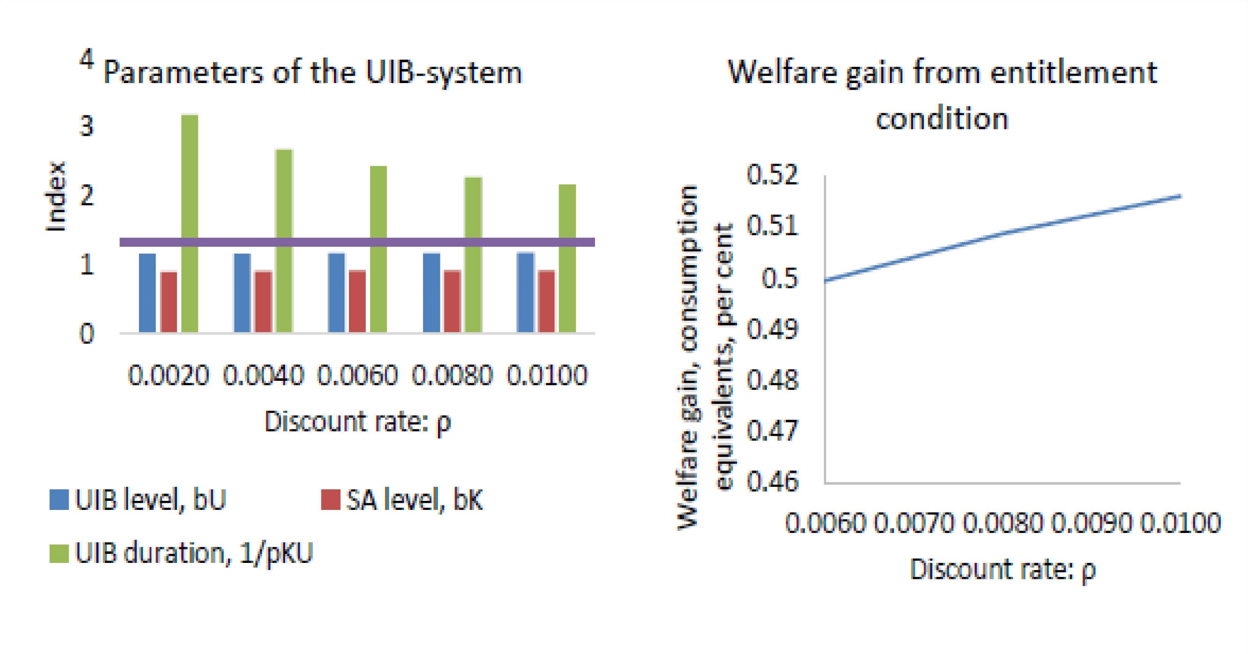
For the entire considered range of the job separation rate, the inclusion of the employment condition has marked effects on the choice of the other dimensions of the unemployment insurance scheme, cf. Figure 6. As in the previous section, benefit duration is more than twice as long, unemployment benefits is up to 20% larger, while social assistance is up to 10% lower when the employment requirement is included and optimally set. The ex ante welfare improvement of including an employment requirement in the UI scheme is rather large for all considered values of the job separation rate, and it ranges from 0.39% (with $p_{U,E} = 0.006$) to 0.58% (with $p_{U,E} = 0.10$) in consumption equivalents.

Figure 6: Variations in the job separation rate ($p_{U,E}$): Characteristics of the UI system and welfare gains



For all considered values of the discount rate (ρ) there are large differences between the optimal UI scheme with and without an employment condition. The ex ante welfare improvement of including an employment condition varies from 0.47% (with $\rho = 0.002$) to 0.52% (with $\rho = 0.010$) in consumption equivalents. Hence, the marked effects of letting unemployment insurance depend on a work requirement is not conditioned on the specific calibration used in the benchmark analysis above.

Figure 7: Variations in the discount rate (ρ): Characteristics of the UI system and welfare gains



5 Concluding remarks

The literature on the optimal design of unemployment insurance scheme largely ignore employment requirements, although they are an important part of eligibility conditions in unemployment insurance schemes, see Venn (2012). In a standard search-matching model we have considered the role of such employment requirements for the incentive structure (search) and optimal design of the unemployment insurance scheme. The employment requirement is shown to have an important role for the incentive structure, and benefit duration and employment requirements are substitute instruments in ensuring job search incentives. Comparing optimal policies with and without the active use of the employment requirement also showed significant effects on benefits levels as well as the benefit duration.

We also explored how the optimal design of the unemployment insurance scheme depends on the need for insurance either due to higher risk aversion or more turbulence in the labour market (higher job separation rate). While either change have marked effects on the optimal design of the UI scheme along all four dimensions (the two benefit levels, benefit duration and employment requirements), the response differ noticeable between the two reasons for more insurance; more risk aversion leads to higher benefit levels, longer benefit duration, but more tight eligibility conditions, whereas a higher job separation rate leads to longer benefit duration and more lax entitlement conditions, but lower benefit levels. Hence, the system does not unambiguously become more generous. Incentives matter, and therefore the improved generosity in some dimensions is balanced by tightening in other dimensions. These findings also stress the interdependencies between all dimensions of the UI system; one dimension cannot be assess independently of the other.

The specific design of the UI system also has implications for the composition of gross unemployment. Introducing employment requirements leads not only to lower gross unemployment, but also to a change in composition with a lower share of SA-unemployed. Since, the UI-unemployed may be termed short-term and SA-unemployed long-term unemployed, it is interesting that the design of the unemployment insurance scheme in this way changes the relative size of the two groups. This points to important distributional effects of entitlement conditions.

The results of the paper show that neglecting employment requirements may lead to very biased recommendations on the optimal design of unemployment insurance schemes. The model framework used is highly stylized to bring forth the role of employment requirements, and there is clearly a need for more work - theoretical and empirical - on the role such requirement play as part of the eligibility conditions for unemployment benefits. Recently, a series of papers have investigated optimal UI systems in search-matching models across the business cycle (e.g. Andersen & Svarer (2010), Mitman & Rabinovich (2015) and Landais et al. (forthcoming)). Neither of these papers explicitly evaluates the optimal response of an eligibility criterion to changes in the business cycle, but given the insights from the current paper it may be a fruitful avenue for future research to investigate the optimal design of an unemployment insurance system across the business cycle where also the eligibility criterion for benefits is allowed to vary.

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Appendix: Search and the properties of the unemployment insurance scheme

The first order conditions determining search can conveniently be written

$$\Lambda_s^U(s_U, b_U, \tau, \alpha, V_E - V_U) \equiv -\frac{\partial g(b_U + \Pi, 1 - s_U)}{\partial s_U} + \alpha [V_E - V_U] = 0$$
$$\Lambda_s^K(s_K, b_K, \tau, \alpha, V_N - V_K) \equiv -\frac{\partial g(b_K + \Pi, 1 - s_K)}{\partial s_K} + \alpha [V_N - V_K] = 0$$

and the associated second order conditions read (sub-indices indicate derivatives wrt. to the variable stated)

$$\Lambda_{ss}^U(\cdot) < 0$$
$$\Lambda_{ss}^K(\cdot) < 0.$$

In the following the benefits levels (b_U, b_K) are given, as are all "macro variables" (τ, α, w_U, w_N), and we are interested in the role of $p_{K,U}$ and $p_{E,N}$. Totally differentiating we find

$$\Lambda_{ss}^U ds_U + \Lambda_{sz}^U dz = 0 \text{ for } z = p_{K,U}, p_{E,N}.$$

It follows that

$$\frac{ds_U}{dz} = -\frac{\Lambda_{sz}^U}{\Lambda_{ss}^U}$$

and hence

$$\text{Sign}\left[\frac{ds_U}{dz}\right] = \text{Sign}\left[\Lambda_{sz}^U\right]$$

where

$$\Lambda_{sz}^U = \alpha \frac{d[V_E - V_U]}{dz}, \text{ with } z = p_{K,U}, p_{E,N}.$$

Similar expressions apply for s_K.

Hence, to clarify how $(p_{K,U}, p_{E,N})$ affect search for U- and K-types we need to know how $V_E - V_U$ and $V_N - V_K$ are affected. Defining the short-hands

$$h_E(\cdot) \equiv h(w_E[1-\tau] + \Pi, 1 - l)$$
$$h_N(\cdot) \equiv h(w_N[1-\tau] + \Pi, 1 - l)$$
$$g_U(\cdot) \equiv g(b_U + \Pi, 1 - s_U)$$
$$g_K(\cdot) \equiv g(b_K + \Pi, 1 - s_K)$$

and using the value functions, we have

$$\rho[V_E - V_U] = h_E(\cdot) - g_U(\cdot) + p_{U,E}[V_U - V_E] - \alpha s_U [V_E - V_U] - p_{K,U}[V_K - V_U]$$
$$\rho[V_N - V_K] = h_N(\cdot) - g_K(\cdot) + p_{U,E}[V_K - V_N] - \alpha s_K [V_N - V_K] + p_{E,N}[V_E - V_N]$$

implying

$$\begin{aligned}
\rho\left[V_E - V_U\right] &= h_E(\cdot) - g_U(\cdot) + p_{U,E}\left[V_U - V_E\right] - \alpha s_U\left[V_E - V_U\right] - p_{K,U}\left[V_K - V_E + V_E - V_U\right] \\
\rho\left[V_N - V_K\right] &= h_N(\cdot) - g_K(\cdot) + p_{U,E}\left[V_K - V_N\right] - \alpha s_K\left[V_N - V_K\right] + p_{E,N}\left[V_E - V_K + V_K - V_N\right]
\end{aligned}$$

and thus

$$\left[V_E - V_U\right] = \frac{h_E(\cdot) - g_U(\cdot) - p_{K,U}\left[V_K - V_E\right]}{\rho + p_{U,E} + \alpha s_U + p_{K,U}} \quad (11)$$

$$\left[V_N - V_K\right] = \frac{h_N(\cdot) - g_K(\cdot) - p_{E,N}\left[V_K - V_E\right]}{\rho + p_{U,E} + \alpha s_K + p_{E,N}}. \quad (12)$$

Using that

$$\rho\left[V_K - V_E\right] = g_K(\cdot) - h_E(\cdot) + \alpha s_K\left[V_N - V_K\right] - p_{U,E}\left[V_U - V_E\right]$$

(11) and (12) implies

$$\begin{aligned}
\left[\rho + p_{U,E} + \alpha s_U + p_{K,U}\right]\left[V_E - V_U\right] &= h_E(\cdot) - g_U(\cdot) - \frac{p_{K,U}}{\rho}\left[g_K(\cdot) - h_E(\cdot)\right. \\
&\quad \left. + \alpha s_K\left[V_N - V_K\right] - p_{U,E}\left[V_U - V_E\right]\right] \\
\left[\rho + p_{U,E} + \alpha s_K + p_{E,N}\right]\left[V_N - V_K\right] &= h_N(\cdot) - g_K(\cdot) - \frac{p_{E,N}}{\rho}\left[g_K(\cdot) - h_E(\cdot)\right. \\
&\quad \left. + \alpha s_K\left[V_N - V_K\right] - p_{U,E}\left[V_U - V_E\right]\right]
\end{aligned}$$

which in turn can be written

$$\begin{aligned}
\left[\rho + p_{U,E} + \alpha s_U + p_{K,U} + \frac{p_{K,U}}{\rho}p_{U,E}\right]\left[V_E - V_U\right] &= h_E(\cdot) - g_U(\cdot) + \frac{p_{K,U}}{\rho}\left[h_E(\cdot) - g_K(\cdot)\right. \\
&\quad \left. - \alpha s_K\left[V_N - V_K\right]\right] \\
\left[\rho + p_{U,E} + \alpha s_K + p_{E,N} + \frac{p_{E,N}}{\rho}\alpha s_K\right]\left[V_N - V_K\right] &= h_N(\cdot) - g_K(\cdot) + \frac{p_{E,N}}{\rho}\left[h_E(\cdot) - g_K(\cdot)\right. \\
&\quad \left. - p_{U,E}\left[V_E - V_U\right]\right].
\end{aligned}$$

Totally differentiating yields

$$\begin{aligned}
\left[V_E - V_U\right](1 + \frac{p_{U,E}}{\rho})dp_{K,U} + A_1 d\left[V_E - V_U\right] &= \frac{1}{\rho}A_2 dp_{K,U} - \frac{p_{K,U}}{\rho}\alpha s_K d\left[V_N - V_K\right] \\
\left[V_N - V_K\right]\left[1 + \frac{1}{\rho}\alpha s_K\right]dp_{E,N} + B_1 d\left[V_N - V_K\right] &= \frac{1}{\rho}B_2 dp_{E,N} - \frac{p_{E,N}}{\rho}p_{U,E} d\left[V_E - V_U\right]
\end{aligned}$$

where

$$\begin{aligned}
A_1 &\equiv \left[\rho + p_{U,E} + \alpha s_U + p_{K,U} + \frac{p_{K,U}}{\rho}p_{U,E}\right] > 0 \\
A_2 &\equiv \left[h_E(\cdot) - g_K(\cdot) - \alpha s_K\left[V_N - V_K\right]\right] \lesseqgtr 0 \\
B_1 &\equiv \rho + p_{U,E} + \alpha s_K + p_{E,N} + \frac{p_{E,N}}{\rho}\alpha s_K > 0 \\
B_2 &\equiv h_E(\cdot) - g_K(\cdot) - p_{U,E}\left[V_E - V_U\right] \lesseqgtr 0.
\end{aligned}$$

Hence,

$$
\begin{aligned}
A_1 d\left[V_E - V_U\right] &= \left[\frac{1}{\rho}A_2 - \left[V_E - V_U\right](1 + \frac{p_{U,E}}{\rho})\right] dp_{K,U} \\
&\quad - \frac{p_{K,U}}{\rho}\alpha s_K d\left[V_N - V_K\right] \qquad (13) \\
B_1 d\left[V_N - V_K\right] &= \left[\frac{1}{\rho}B_2 - \left[V_N - V_K\right]\left[1 + \frac{1}{\rho}\alpha s_K\right]\right] dp_{E,N} \\
&\quad - \frac{p_{E,N}}{\rho} p_{U,E} d\left[V_E - V_U\right]. \qquad (14)
\end{aligned}
$$

Before proceeding we prove that

$$\frac{1}{\rho}B_2 - \left[V_N - V_K\right]\left[1 + \frac{1}{\rho}\alpha s_K\right] > 0$$

or

$$h_E(\cdot) - g_K(\cdot) - p_{U,E}\left[V_E - V_U\right] > \left[V_N - V_K\right]\left[\rho + \alpha s_K\right].$$

We have from the value functions that

$$\left[\rho + \alpha s_K\right]\left[V_N - V_K\right] = h_N(\cdot) - g_K(\cdot) + p_{U,E}\left[V_K - V_N\right] + p_{E,N}\left[V_E - V_N\right]$$

and hence, the inequality can be rewritten

$$h_E(\cdot) - h_N(\cdot) - p_{U,E}\left[V_E - V_U\right] > p_{U,E}\left[V_K - V_N\right] + p_{E,N}\left[V_E - V_N\right].$$

Using that

$$\rho\left[V_E - V_N\right] = h_E(\cdot) - h_N(\cdot) + p_{U,E}\left[V_U - V_E\right] - p_{U,E}\left[V_K - V_N\right] - p_{E,N}\left[V_E - V_N\right]$$

we have that the inequality reduces to

$$\rho\left[V_E - V_N\right] > 0$$

which is fulfilled.

We also prove that $\frac{1}{\rho}A_2 - \left[V_E - V_U\right](1 + \frac{p_{U,E}}{\rho}) > 0$ or

$$h_E(\cdot) - g_K(\cdot) - \alpha s_K\left[V_N - V_K\right] > \left[V_E - V_U\right](\rho + p_{U,E}).$$

We have from the value functions that

$$(\rho + p_{U,E})\left[V_E - V_U\right] = h_E(\cdot) - g_U(\cdot) - \alpha s_U\left[V_E - V_U\right] - p_{K,U}\left[V_K - V_U\right]$$

and hence, the inequality can be written

$$g_U(\cdot) - g_K(\cdot) > \alpha s_K [V_N - V_K] - \alpha s_U [V_E - V_U] - p_{K,U} [V_K - V_U].$$

Using that

$$\rho [V_U - V_K] = g_U(\cdot) - g_K(\cdot) + \alpha s_U [V_E - V_U] + p_{K,U} [V_K - V_U] - \alpha s_K [V_N - V_K]$$

the inequality reduces to

$$\rho [V_U - V_K] > 0$$

which is fulfilled.

Finally, also note that

$$B_1 - \frac{p_{E,N}}{\rho} p_{U,E} \frac{p_{K,U}}{\rho} \frac{\alpha s_K}{A_1} = \frac{1}{A_1} \left[A_1 B_1 - \frac{p_{E,N}}{\rho} p_{U,E} \frac{p_{K,U}}{\rho} \alpha s_K \right] > 0.$$

Returning to (13) and (14) we have for $dp_{K,U} = 0$ that

$$A_1 d [V_E - V_U] = -\frac{p_{K,U}}{\rho} \alpha s_K d [V_N - V_K]$$

$$\left[B_1 - \frac{p_{E,N}}{\rho} p_{U,E} \frac{p_{K,U}}{\rho} \frac{\alpha s_K}{A_1} \right] d [V_N - V_K] = \left[\frac{1}{\rho} B_2 - [V_N - V_K] \left[1 + \frac{1}{\rho} \alpha s_K \right] \right] dp_{E,N}$$

$$\begin{aligned} d [V_N - V_K] &= \frac{\left[\frac{1}{\rho} B_2 - [V_N - V_K] \left[1 + \frac{1}{\rho} \alpha s_K \right] \right]}{\left[B_1 - \frac{p_{E,N}}{\rho} p_{U,E} \frac{p_{K,U}}{\rho} \frac{\alpha s_K}{A_1} \right]} dp_{E,N} \\ d [V_E - V_U] &= -\frac{1}{A_1} \frac{p_{K,U}}{\rho} \alpha s_K \frac{\left[\frac{1}{\rho} B_2 - [V_N - V_K] \left[1 + \frac{1}{\rho} \alpha s_K \right] \right]}{\left[B_1 - \frac{p_{E,N}}{\rho} p_{U,E} \frac{p_{K,U}}{\rho} \frac{\alpha s_K}{A_1} \right]} dp_{E,N} \\ \frac{d [V_N - V_K]}{d [V_E - V_U]} &= \left[-\frac{1}{A_1} \frac{p_{K,U}}{\rho} \alpha s_K \right]^{-1} \end{aligned}$$

$$\begin{aligned} \frac{d [V_N - V_K]}{dp_{E,N}} &= \frac{\left[\frac{1}{\rho} B_2 - [V_N - V_K] \left[1 + \frac{1}{\rho} \alpha s_K \right] \right]}{\left[B_1 - \frac{p_{E,N}}{\rho} p_{U,E} \frac{p_{K,U}}{\rho} \frac{\alpha s_K}{A_1} \right]} \\ \frac{d [V_E - V_U]}{dp_{E,N}} &= -\frac{1}{A_1} \frac{p_{K,U}}{\rho} \alpha s_K \frac{\left[\frac{1}{\rho} B_2 - [V_N - V_K] \left[1 + \frac{1}{\rho} \alpha s_K \right] \right]}{\left[B_1 - \frac{p_{E,N}}{\rho} p_{U,E} \frac{p_{K,U}}{\rho} \frac{\alpha s_K}{A_1} \right]}. \end{aligned}$$

and for $dp_{E,N} = 0$ that

$$B_1 d\left[V_N - V_K\right] = -\frac{p_{E,N}}{\rho} p_{U,E} d\left[V_E - V_U\right]$$

$$\left[A_1 - \frac{p_{E,N}}{\rho} p_{U,E} \frac{p_{K,U}}{\rho} \frac{\alpha s_K}{B_1}\right] d\left[V_E - V_U\right] = \left[\frac{1}{\rho} A_2 - \left[V_E - V_U\right](1 + \frac{p_{U,E}}{\rho})\right] dp_{K,U}$$

$$\begin{aligned} d\left[V_E - V_U\right] &= \frac{\left[\frac{1}{\rho} A_2 - \left[V_E - V_U\right](1 + \frac{p_{U,E}}{\rho})\right]}{\left[A_1 - \frac{p_{E,N}}{\rho} p_{U,E} \frac{p_{K,U}}{\rho} \frac{\alpha s_K}{B_1}\right]} dp_{K,U} \\ d\left[V_N - V_K\right] &= \frac{-1}{B_1} \frac{p_{E,N}}{\rho} p_{U,E} \frac{\left[\frac{1}{\rho} A_2 - \left[V_E - V_U\right](1 + \frac{p_{U,E}}{\rho})\right]}{\left[A_1 - \frac{p_{E,N}}{\rho} p_{U,E} \frac{p_{K,U}}{\rho} \frac{\alpha s_K}{B_1}\right]} dp_{K,U} \\ \frac{d\left[V_N - V_K\right]}{d\left[V_E - V_U\right]} &= \frac{-1}{B_1} \frac{p_{E,N}}{\rho} p_{U,E} \end{aligned}$$

$$\begin{aligned} \frac{d\left[V_E - V_U\right]}{dp_{K,U}} &= \frac{\left[\frac{1}{\rho} A_2 - \left[V_E - V_U\right](1 + \frac{p_{U,E}}{\rho})\right]}{\left[A_1 - \frac{p_{E,N}}{\rho} p_{U,E} \frac{p_{K,U}}{\rho} \frac{\alpha s_K}{B_1}\right]} \\ \frac{d\left[V_N - V_K\right]}{dp_{K,U}} &= \frac{-1}{B_1} \frac{p_{E,N}}{\rho} p_{U,E} \frac{\left[\frac{1}{\rho} A_2 - \left[V_E - V_U\right](1 + \frac{p_{U,E}}{\rho})\right]}{\left[A_1 - \frac{p_{E,N}}{\rho} p_{U,E} \frac{p_{K,U}}{\rho} \frac{\alpha s_K}{B_1}\right]}. \end{aligned}$$

Hence, we have established the following signs

	$d\left[V_N - V_K\right]$	$d\left[V_E - V_U\right]$
$dp_{E,N}$	> 0	< 0
$dp_{K,U}$	< 0	> 0

.

This implies that

$$\begin{aligned} \frac{ds_U}{dp_{K,U}} &> 0 \;\; ; \;\; \frac{ds_K}{dp_{K,U}} < 0 \\ \frac{ds_U}{dp_{E,N}} &< 0 \;\; ; \;\; \frac{ds_K}{dp_{E,N}} > 0. \end{aligned}$$

Marginal rates of substitution

Consider next the marginal rates of return, i.e., combinations of $p_{K,U}$ and $p_{E,N}$ leaving $V_E - V_U$ and thus search s_U unchanged (and similarly for s_K). Using

$$\begin{aligned} A_1 d\left[V_E - V_U\right] &= \left[\frac{1}{\rho} A_2 - \left[V_E - V_U\right](1 + \frac{p_{U,E}}{\rho})\right] dp_{K,U} - \frac{p_{K,U}}{\rho} \alpha s_K d\left[V_N - V_K\right] \\ B_1 d\left[V_N - V_K\right] &= \left[\frac{1}{\rho} B_2 - \left[V_N - V_K\right]\left[1 + \frac{1}{\rho} \alpha s_K\right]\right] dp_{E,N} - \frac{p_{E,N}}{\rho} p_{U,E} d\left[V_E - V_U\right] \end{aligned}$$

and imposing $d\left[V_E - V_U\right] = 0$, we obtain

$$0 = \left[\frac{1}{\rho}A_2 - \left[V_E - V_U\right](1 + \frac{p_{U,E}}{\rho})\right] dp_{K,U} - \frac{p_{K,U}}{\rho}\frac{\alpha s_K}{B_1}\left[\frac{1}{\rho}B_2 - \left[V_N - V_K\right]\left[1 + \frac{1}{\rho}\alpha s_K\right]\right] dp_{E,N}$$

and hence,

$$\frac{dp_{K,U}}{dp_{E,N}}\mid_{[V_E - V_U]=\text{constant}} = \frac{\frac{p_{K,U}}{\rho}\frac{\alpha s_K}{B_1}\left[\frac{1}{\rho}B_2 - \left[V_N - V_K\right]\left[1 + \frac{1}{\rho}\alpha s_K\right]\right]}{\left[\frac{1}{\rho}A_2 - \left[V_E - V_U\right](1 + \frac{p_{U,E}}{\rho})\right]} > 0.$$

Similarly, for s_K where we have that $d\left[V_N - V_K\right] = 0$ implies

$$0 = \left[\frac{1}{\rho}B_2 - \left[V_N - V_K\right]\left[1 + \frac{1}{\rho}\alpha s_K\right]\right] dp_{E,N} - \frac{p_{E,N}}{\rho}\frac{p_{U,E}}{A_1}\left[\frac{1}{\rho}A_2 - \left[V_E - V_U\right](1 + \frac{p_{U,E}}{\rho})\right] dp_{K,U}$$

and hence,

$$\frac{dp_{E,N}}{dp_{K,U}}\mid_{[V_N - V_K]=\text{const}} = \frac{\frac{p_{E,N}}{\rho}\frac{p_{U,E}}{A_1}\left[\frac{1}{\rho}A_2 - \left[V_E - V_U\right](1 + \frac{p_{U,E}}{\rho})\right]}{\left[\frac{1}{\rho}B_2 - \left[V_N - V_K\right]\left[1 + \frac{1}{\rho}\alpha s_K\right]\right]} > 0.$$

Note that (recall that $A_1 > 0$ and $B_1 > 0$)

$$\frac{dp_{K,U}}{dp_{E,N}}\mid_{[V_E - V_U]=\text{constant}} \frac{dp_{E,N}}{dp_{K,U}}\mid_{[V_N - V_K]=\text{const}} = \frac{p_{K,U}}{\rho}\frac{\alpha s_K}{B_1}\frac{p_{E,N}}{\rho}\frac{p_{U,E}}{A_1} \lesseqgtr 1.$$

From the envelope theorem we know that the utility effect of a given policy change is given by the direct utility effects (all indirect effects via behavior wash out via first order conditions).

	$d\left[V_N - V_K\right]$	$d\left[V_E - V_U\right]$
$dp_{E,N}$	> 0	< 0
$dp_{K,U}$	< 0	> 0

and

$$\begin{aligned}
\rho V_E &= h\left(w\left[1-\tau\right] + \Pi, 1 - l\right) + p_{U,E}\left[V_U - V_E\right] \\
\rho V_N &= h\left(w\left[1-\tau\right] + \Pi, 1 - l\right) + p_{U,E}\left[V_K - V_N\right] + p_{E,N}\left[V_E - V_N\right] \\
\rho V_U &= g\left(b_U + \Pi, 1 - s_U\right) + \alpha s_U\left[V_E - V_U\right] + p_{K,U}\left[V_K - V_U\right] \\
\rho V_K &= g\left(b_K + \Pi, 1 - s_K\right) + \alpha s_K\left[V_N - V_K\right]
\end{aligned}$$

we thus have: i) an increase in $p_{K,U}$ leads to a decrease in V_E and a decrease in V_K, while there is an ambiguous effect on V_N and V_U, ii) an increase in $p_{E,N}$ leads to an increase in V_E and an increase in V_K, while there is an ambiguous effect on V_N and V_U.

Iso-gross unemployment loci

Note first that we have

$$u = \frac{p_{E,N}}{p_{K,U}} n$$

implying

$$k = \frac{p_{U,E} + p_{E,N}}{\alpha s_K} n = \frac{p_{U,E} + p_{E,N}}{\alpha s_K} \frac{p_{K,U}}{p_{E,N}} u$$

and

$$u = \frac{p_{U,E}}{\alpha s_U + p_{K,U} + p_{U,E} + p_{U,E} \frac{p_{U,E}+p_{E,N}}{\alpha s_K} \frac{p_{K,U}}{p_{E,N}} + p_{U,E} \frac{p_{K,U}}{p_{E,N}}}$$

$$\begin{aligned}
e &= 1 - u - n - k \\
&= 1 - (1 - \frac{p_{K,U}}{p_{E,N}} - \frac{p_{U,E} + p_{E,N}}{\alpha s_K} \frac{p_{K,U}}{p_{E,N}}) \frac{p_{U,E}}{\alpha s_U + p_{K,U} + p_{U,E} + p_{U,E} \frac{p_{U,E}+p_{E,N}}{\alpha s_K} \frac{p_{K,U}}{p_{E,N}} + p_{U,E} \frac{p_{K,U}}{p_{E,N}}} \\
&= 1 - (\frac{p_{E,N} \alpha s_K - (\alpha s_K + p_{U,E} + p_{E,N}) p_{K,U}}{\alpha s_K p_{E,N}}) \\
&\quad \times \frac{p_{U,E}}{\alpha s_U + p_{K,U} + p_{U,E} + p_{U,E} \frac{p_{U,E}+p_{E,N}}{\alpha s_K} \frac{p_{K,U}}{p_{E,N}} + p_{U,E} \frac{p_{K,U}}{p_{E,N}}}.
\end{aligned}$$

It follows that gross unemployment $(u + k)$ is given as

$$\begin{aligned}
u + k &= u \left[1 + \frac{p_{U,E} + p_{E,N}}{\alpha s_K} \frac{p_{K,U}}{p_{E,N}} \right] \\
&= \frac{p_{U,E} \left[1 + \frac{p_{U,E}+p_{E,N}}{\alpha s_K} \frac{p_{K,U}}{p_{E,N}} \right]}{\alpha s_U + p_{K,U} + p_{U,E} + p_{U,E} \frac{p_{U,E}+p_{E,N}}{\alpha s_K} \frac{p_{K,U}}{p_{E,N}} + p_{U,E} \frac{p_{K,U}}{p_{E,N}}} < 1
\end{aligned}$$

or

$$\begin{aligned}
\frac{1}{u+k} &= 1 + \frac{\alpha s_U + p_{K,U} + p_{U,E} \frac{p_{K,U}}{p_{E,N}}}{p_{U,E} + \frac{p_{U,E}+p_{E,N}}{\alpha s_K} \frac{p_{K,U}}{p_{E,N}} p_{U,E}} > 1 \\
&= 1 + \frac{\alpha s_U + p_{K,U} + \frac{p_{U,E}}{p_{E,N}} p_{K,U}}{p_{U,E} + \frac{\frac{p_{U,E}}{p_{E,N}} + 1}{\alpha s_K} p_{K,U} p_{U,E}}.
\end{aligned}$$

Where

$$\begin{aligned}
\frac{\partial}{\partial s_U} \left(\frac{1}{u+k} \right) &= \frac{\alpha}{p_{U,E} + \frac{\frac{p_{U,E}}{p_{E,N}} + 1}{\alpha s_K} p_{K,U} p_{U,E}} > 0 \\
\frac{\partial}{\partial s_K} \left(\frac{1}{u+k} \right) &= \frac{\alpha s_U + p_{K,U} + \frac{p_{U,E}}{p_{E,N}} p_{K,U}}{\left[p_{U,E} + \frac{\frac{p_{U,E}}{p_{E,N}} + 1}{\alpha s_K} p_{K,U} p_{U,E} \right]^2} \left[\alpha \frac{\frac{p_{U,E}}{p_{E,N}} + 1}{[\alpha s_K]^2} p_{K,U} p_{U,E} \right] > 0
\end{aligned}$$

$$\frac{\partial}{\partial p_{EN}}\left(\frac{1}{u+k}\right) = \frac{p_{K,U}\left[p_{U,E}+\frac{\frac{p_{U,E}}{p_{E,N}}+1}{\alpha s_K}p_{K,U}p_{U,E}\right] - \left[\alpha s_U + p_{K,U} + \frac{p_{U,E}}{p_{E,N}}p_{K,U}\right]\left[\frac{p_{K,U}p_{U,E}}{\alpha s_K}\right]}{\left[p_{U,E}+\frac{\frac{p_{U,E}}{p_{E,N}}+1}{\alpha s_K}p_{K,U}p_{U,E}\right]^2}\left(-\frac{p_{U,E}}{p_{E,N}^2}\right)$$

$$= \frac{p_{K,U}\left[p_{U,E}+\frac{1}{\alpha s_K}p_{K,U}p_{U,E}\right] - \left[\alpha s_U + p_{K,U}\right]\left[\frac{p_{K,U}p_{U,E}}{\alpha s_K}\right]}{\left[p_{U,E}+\frac{\frac{p_{U,E}}{p_{E,N}}+1}{\alpha s_K}p_{K,U}p_{U,E}\right]^2}\left(-\frac{p_{U,E}}{p_{E,N}^2}\right)$$

$$= \frac{p_{K,U}p_{U,E}\left[1-\frac{s_U}{s_K}\right]}{\left[p_{U,E}+\frac{\frac{p_{U,E}}{p_{E,N}}+1}{\alpha s_K}p_{K,U}p_{U,E}\right]^2}\left(-\frac{p_{U,E}}{p_{E,N}^2}\right) \lesseqgtr 0 \quad \text{for } \frac{s_U}{s_K} \lesseqgtr 1$$

$$\frac{\partial}{\partial p_{KU}}\left(\frac{1}{u+k}\right) = \frac{\left[1+\frac{p_{U,E}}{p_{E,N}}\right]\left[p_{U,E}+\frac{\frac{p_{U,E}}{p_{E,N}}+1}{\alpha s_K}p_{K,U}p_{U,E}\right] - \left[\alpha s_U + p_{K,U} + \frac{p_{U,E}}{p_{E,N}}p_{K,U}\right]\left[\frac{\frac{p_{U,E}}{p_{E,N}}+1}{\alpha s_K}p_{U,E}\right]}{\left[p_{U,E}+\frac{\frac{p_{U,E}}{p_{E,N}}+1}{\alpha s_K}p_{K,U}p_{U,E}\right]^2}$$

$$= \frac{\left[1+\frac{p_{U,E}}{p_{E,N}}\right]\left[p_{U,E}+\frac{p_{K,U}p_{U,E}}{\alpha s_K}\right] - \left[\alpha s_U + p_{K,U}\right]\left[\frac{\frac{p_{U,E}}{p_{E,N}}+1}{\alpha s_K}p_{U,E}\right]}{\left[p_{U,E}+\frac{\frac{p_{U,E}}{p_{E,N}}+1}{\alpha s_K}p_{K,U}p_{U,E}\right]^2}$$

$$= \frac{p_{U,E} - \alpha s_U \frac{1}{\alpha s_K}p_{U,E}}{\left[p_{U,E}+\frac{\frac{p_{U,E}}{p_{E,N}}+1}{\alpha s_K}p_{K,U}p_{U,E}\right]^2}\left[1+\frac{p_{U,E}}{p_{E,N}}\right] < 0$$

$$= \frac{p_{U,E}\left(1-\frac{s_U}{s_K}\right)\left[1+\frac{p_{U,E}}{p_{E,N}}\right]}{\left[p_{U,E}+\frac{\frac{p_{U,E}}{p_{E,N}}+1}{\alpha s_K}p_{K,U}p_{U,E}\right]^2} \lesseqgtr 0 \quad \text{for } \frac{s_U}{s_K} \gtreqless 1.$$

Hence, the gross unemployment can be written in implicit form as

$$u + k = F(s_U(p_{K,U}, p_{E,N}), s_K(p_{K,U}, p_{E,N}), p_{K,U}, p_{E,N})$$

Note that

$$sign\frac{\partial F(\cdot)}{\partial z} = -sign\frac{\partial}{\partial z}\left(\frac{1}{u+k}\right).$$

It follows that

$$\begin{aligned}
\frac{\partial F(\cdot)}{\partial s_U} &< 0 \ ; \ \frac{\partial F(\cdot)}{\partial s_K} < 0 \\
sign\left(\frac{\partial F(\cdot)}{\partial p_{K,U}}\right) &= sign(s_U - s_K) \\
sign\left(\frac{\partial F(\cdot)}{\partial p_{E,N}}\right) &= -sign\left(\frac{\partial F(\cdot)}{\partial p_{K,U}}\right).
\end{aligned}$$

We have that

$$\begin{aligned}
d(u+k) &= \frac{\partial F(\cdot)}{\partial s_U} ds_U + \frac{\partial F(\cdot)}{\partial s_K} ds_K + \frac{\partial F(\cdot)}{\partial p_{K,U}} dp_{K,U} + \frac{\partial F(\cdot)}{\partial p_{E,N}} dp_{E,N} \\
&= \frac{\partial F(\cdot)}{\partial s_U}\left[\frac{\partial s_U}{\partial p_{K,U}} dp_{K,U} + \frac{\partial s_U}{\partial p_{E,N}} dp_{E,N}\right] + \frac{\partial F(\cdot)}{\partial s_K}\left[\frac{\partial s_K}{\partial p_{K,U}} dp_{K,U} + \frac{\partial s_K}{\partial p_{E,N}} dp_{E,N}\right] \\
&\quad + \frac{\partial F(\cdot)}{\partial p_{K,U}} dp_{K,U} + \frac{\partial F(\cdot)}{\partial p_{E,N}} dp_{E,N}.
\end{aligned}$$

For $d(u+k) = 0$ we have

$$\begin{aligned}
&-\left[\frac{\partial F(\cdot)}{\partial p_{E,N}} + \frac{\partial s_U}{\partial p_{E,N}}\frac{\partial F(\cdot)}{\partial s_U} + \frac{\partial s_K}{\partial p_{E,N}}\frac{\partial F(\cdot)}{\partial s_K}\right] dp_{E,N} \\
&= \left[\frac{\partial F(\cdot)}{\partial p_{K,U}} + \frac{\partial s_K}{\partial p_{K,U}}\frac{\partial F(\cdot)}{\partial s_K} + \frac{\partial s_U}{\partial p_{K,U}}\frac{\partial F(\cdot)}{\partial s_U}\right] dp_{K,U} \\
\frac{dp_{E,N}}{dp_{K,U}} &= -\frac{\frac{\partial F(\cdot)}{\partial p_{K,U}} + \frac{\partial s_K}{\partial p_{K,U}}\frac{\partial F(\cdot)}{\partial s_K} + \frac{\partial s_U}{\partial p_{K,U}}\frac{\partial F(\cdot)}{\partial s_U}}{\frac{\partial F(\cdot)}{\partial p_{E,N}} + \frac{\partial s_U}{\partial p_{E,N}}\frac{\partial F(\cdot)}{\partial s_U} + \frac{\partial s_K}{\partial p_{E,N}}\frac{\partial F(\cdot)}{\partial s_K}}. \qquad (15)
\end{aligned}$$

Notice that

$$\begin{aligned}
\frac{ds_U}{dp_{K,U}} &> 0; \frac{ds_K}{dp_{K,U}} < 0 \\
\frac{ds_U}{dp_{E,N}} &< 0; \frac{ds_K}{dp_{E,N}} > 0.
\end{aligned}$$

Hence, the numerator and denominator of (15) are in general ambiguously signed.