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Nonlinear state space model identification of synchronous generators

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Abstract

A method for identification of a synchronous generator is suggested in this paper. The method uses the theoretical relations of machine parameters and the Prony method to find the state space model of the system. Such models are useful for controller design and stability tests. The proposed identification method is applied to a third order model of a synchronous generator. In this study, the field voltage is considered as the input and the active output power and the rotor angle are considered as the outputs of the synchronous generator. Simulation results show good accuracy of the identified model.

Keywords: Identification; Synchronous generators; Nonlinear model; State space model; Prony method; Regression equation

1. Introduction

The increasing complexity of modern power systems highlights the need for advanced identification techniques. The identified model is used to design an effective controller. Obviously, more accurate model leads to more accurate controller design and satisfactory results are obtained [1].

Among the power system elements, synchronous generators play an important role in the stability of the power systems. A proper model for synchronous generators is essential for a valid analysis of stability and dynamic performance.

There are three approaches for modeling the synchronous generators [2]: white-box modeling [3,4], grey-box modeling [5–7] and black-box modeling [8–13]. The first method is based on off-line tests and the second and the third methods are based on on-line measurements.

The white-box modeling refers to some methods such as the traditional methods of modeling the synchronous generators [3,4]. These methods assume a known structure for the synchronous machine. They address the problem of finding the parameters of the assumed structure. Usually the procedures involve difficult and time-consuming tests, such as short-circuit tests, stand-still frequency response (SSFR) and open circuit frequency response (OCFR) tests. These tests can be carried out when the machine is not in service. This is the main disadvantage of these methods.

To overcome this disadvantage, identification methods based on on-line measurements have gained attention during recent years [5–13].

In the grey-box modeling, one assumes a known structure for the synchronous machine, as the traditional methods, and then the physical parameters are estimated from on-line measurements. In black-box modeling, the structure of the model is not assumed to be known a priori. The only concern is to map the input data set to the output data set.

In the last two decades, there have been significant advances in modern control theory. Techniques that can take in to consideration the plant uncertainties such as LMI, H_∞ and μ-synthesis have gained attention [14,15]. Due to the complexities of these approaches, a low order model which consideres the most important dynamics of the system, is very useful for controller design. Some papers

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refer to the system order reduction to make it more useful for practical purposes [16], but the order reduction method decreases the controller performance in some aspects.

In this paper, the aim is to identify a grey-box model for a synchronous generator. Such a model can be used for system analysis and controller design. The model is based on the theoretical relations between the synchronous machine parameters, so it consideres the physical meaning of each parameter.

To avoid complexity, the known nonlinear model of order three is assumed for the system. The model is quite accurate for studying low-frequency oscillations and stability of power systems. It has also been successfully used for designing classical power system stabilizers (PSSs), too [7].

There are some obvious nonlinear terms in the state space synchronous generator model of order three. In this paper, these nonlinearities are considered and the unknown related parameters are identified.

Most of the previous works done on the topic of synchronous generator parameter estimation [5–7], can only be applied to the linear system model parameter estimation. In ref. [2] a method is suggested for the synchronous machine parameter estimation, which results in some nonlinear set of equations. In each test, the nonlinear set should be solved numerically. Furthermore, the algorithm does not discuss the conditions in which the system of equations can be solved. In some situations, the algorithm may result in a singular set of equations and no solution can be found.

The algorithm derived here to extract nonlinear state space model of a synchronous generator, is a straight forward algorithm for implementation. It requires online measurement of signals with the generator in service. It uses the theoretical background of the state space model and directly finds the nonlinear model. Furthermore, a parametric solution for each parameter is derived. At last, three regression equations, based on online measurements, are needed to find the model, so it is a simple method for implementation.

The paper is organized as follows: the study system used in this paper and the identification method are described in Sections 2 and 3, respectively. The method is applied to a simulated model of a synchronous generator in Section 4. Section 5 concludes the paper.

2. The synchronous generator model

A synchronous generator connected to an infinite bus through a transmission line (Fig. 1), is considered as the study system.

It is assumed that the field voltage, rotor angle and the electrical power can be measured. The procedure for rotor angle measurement can be found in ref. [17].

Applying a small perturbation to the field voltage, the above mentioned signals are measured and used in an algorithm to identify the third order model parameters.

The third order nonlinear model derived in refs. [18,19] is used in this paper. All the parameters are assumed to be defined in per unit values. The model is described by the following nonlinear equations:

$$\begin{aligned}\dot{\delta} &= \omega \\ \dot{\omega} &= \frac{1}{J}(T_{\mathrm{m}} - T_{\mathrm{e}} - D\omega) \\ \dot{e}'_{\mathrm{q}} &= \frac{1}{T'_{\mathrm{do}}}(E_{\mathrm{FD}} - e'_{\mathrm{q}} - (x_{\mathrm{d}} - x'_{\mathrm{d}})i_{\mathrm{d}})\end{aligned} \tag{1}$$

where

$$i_{\mathrm{d}} = \frac{e'_{\mathrm{q}} - V\cos\delta}{x'_{\mathrm{d}}} \tag{2}$$

$$i_{\mathrm{q}} = \frac{V\sin\delta}{x_{\mathrm{q}}} \tag{3}$$

$$T_{\mathrm{e}} = P_{\mathrm{e}} \cong \frac{V}{x'_{\mathrm{d}}}e'_{\mathrm{q}}\sin(\delta) + \frac{V^2}{2}\left(\frac{1}{x_{\mathrm{q}}} - \frac{1}{x'_{\mathrm{d}}}\right)\sin(2\delta) \tag{4}$$

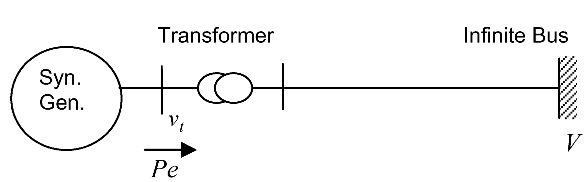

Fig. 1. Structure of the study system.

This model of synchronous generators is very useful for stability analysis and controller design [20–22]. In this model, the influence of magnetic saturation is neglected, therefore, one could assume x_d, x_q and x'_d to be constant. x_d, x_q and x'_d used in this model are the augmented reactances, i.e. the line and transformer reactances are added with them [20,21].

The details of the above model can be found in refs. [18,19]. Some information about the parameter definition is given in Appendix A.

The system states and inputs are defined as:

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \delta \\ \omega \\ e'_q \end{bmatrix}, \qquad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} E_{FD} \\ T_m \end{bmatrix} \tag{5}$$

So, the state space nonlinear model of the system will be as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\dfrac{D}{J} & 0 \\ 0 & 0 & -\dfrac{1}{T'_{do}}\left(\dfrac{x_d}{x'_d}\right) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \dfrac{1}{J} \\ \dfrac{1}{T'_{do}} & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -\dfrac{1}{J}\left(\dfrac{V}{x'_d}x_3 \sin x_1 + \dfrac{V^2}{2}\left(\dfrac{1}{x_q} - \dfrac{1}{x'_d}\right)\sin 2x_1\right) \\ \dfrac{1}{T'_{do}}\left(\dfrac{x_d - x'_d}{x'_d}\right) V \cos x_1 \end{bmatrix} \tag{6}$$

The above model can be summarized as follows:

$$\dot{x} = Ax + Bu + f(x) \tag{7}$$

The linearized model of (7) in the vicinity of an operating point 'o' would be:

$$\Delta\dot{x} = A\,\Delta x + B\,\Delta u + \Delta f(x) \tag{8}$$

In the above equation, A and B will remain the same as in (7). $\Delta f(x)$ would add some terms to the A and B matrixes. Finally, the following model is achieved:

$$\Delta\dot{x} = A'\,\Delta x + B'\,\Delta u \tag{9}$$

According to the special format of the synchronous generator model, the linearized form of $f(x)$ will add some terms in the cells of A matrix which are zero at the basic nonlinear model of (6).

Assuming the generator rotor angle and the electrical power as the outputs, the linearized form of the above model is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\dfrac{K_1}{J} & -\dfrac{D}{J} & -\dfrac{K_2}{J} \\ -\dfrac{K_4}{T'_{do}} & 0 & -\dfrac{1}{K_3 T'_{do}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \dfrac{1}{J} \\ \dfrac{1}{T'_{do}} & 0 \end{bmatrix} \begin{bmatrix} u1 \\ u2 \end{bmatrix}$$
$$y = \begin{bmatrix} \Delta\delta \\ \Delta P_e \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ K_1 & 0 & K_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{10}$$

where

$$K_1 = \frac{V}{x'_d} x_{3o} \cos x_{1o} + V^2\left(\frac{1}{x_q} - \frac{1}{x'_d}\right)\cos 2x_{1o} \tag{11}$$

$$K_2 = \frac{V}{x'_d} \sin x_{1o} \tag{12}$$

$$K_3 = \frac{1}{1 + (x_d - x'_d)/x'_d} = \frac{x'_d}{x_d} \tag{13}$$

$$K_4 = (x_d - x'_d)\frac{V \sin x_{1o}}{x'_d} \tag{14}$$

To avoid complexity in the equations and formulas in the succeeding sections, in the model of Eq. (10), the unknown parameters are defined as $p_1, \ldots, p_6$ and J. On the other hand, the goal is to find a practical approach for synchronous generator model identification, so only the field voltage which can be perturbed more easily than the mechanical torque is considered as the input to the system and the mechanical power is assumed to be constant [2]. Finally, the linear model is written as:

$$\begin{aligned} \dot{x} &= Ax + Bu = \begin{bmatrix} 0 & 1 & 0 \\ p_1 & p_2 & p_3 \\ p_4 & 0 & p_5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ p_6 \end{bmatrix} u \\ y &= Cx = \begin{bmatrix} 1 & 0 & 0 \\ -Jp_1 & 0 & -Jp_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \end{aligned} \tag{15}$$

The coresponding nonlinear model will be:

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & p_2 & 0 \\ 0 & 0 & p_5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ p_6 \end{bmatrix} u + \begin{bmatrix} 0 \\ \dfrac{1}{J}\left(P_{\mathrm{m}} - \dfrac{V}{x'_{\mathrm{d}}} x_3 \sin x_1 - \dfrac{V^2}{2}\left(\dfrac{1}{x_{\mathrm{q}}} - \dfrac{1}{x'_{\mathrm{d}}}\right) \sin 2x_1\right) \\ -(p_5 + p_6)\cos x_1 \end{bmatrix} \\ y &= \begin{bmatrix} x_1 \\ \dfrac{V}{x'_{\mathrm{d}}} x_3 \sin x_1 + \dfrac{V^2}{2}\left(\dfrac{1}{x_{\mathrm{q}}} - \dfrac{1}{x'_{\mathrm{d}}}\right) \sin 2x_1 \end{bmatrix} \end{aligned} \tag{16}$$

Obviously, the above model is a nonlinear model. In ref. [2] an approach is presented which needs to solve a set of nonlinear equations numerically, to find the unknown parameters of the above model. In this study, we would like to introduce an analytic, easy-implemented, direct approach.

It is obvious from (15)–(16) that the linear and the nonlinear models have some common parameteres, which does not depend on the operating point values. Here, first the linear model parameters are estimated, then the remaining nonlinear model parameters are obtained.

3. Identification method

The identification method has three parts, first, the discrete part, second, the Prony approach for eigenvalue computation and third, the steady state response analysis. Each part results in a regression equation. In this section, these parts are described.

3.1. Discrete state space model

Find the discrete form of the system given in (15) by the sampling rate of T. Due to ref. [23], the model changes to:

$$\begin{aligned} X(k+1) &= \Phi X(k) + \Psi u(k) \\ y(k) &= CX(k) \end{aligned} \tag{17}$$

where

$$\Phi = I + AT + \frac{A^2T^2}{2} + \cdots \tag{18}$$

and

$$\Psi = \left(I + \frac{AT}{2} + \frac{A^2T^2}{6} + \cdots\right) BT \tag{19}$$

As we should choose T small enough to have a valid sampled data system, the approximation $T^{\mathrm{m}} \to 0$ for $m \geq 3$ looks as a logic assumption. Using this approximation, Φ and Ψ are given in (20)–(21):

$$\Phi = \begin{bmatrix} 1 + p_1\dfrac{T^2}{2} & T + p_2\dfrac{T^2}{2} & p_3\dfrac{T^2}{2} \\ p_1T + p_1p_2\dfrac{T^2}{2} + p_3p_4\dfrac{T^2}{2} & 1 + p_2T + p_1\dfrac{T^2}{2} + p_2^2\dfrac{T^2}{2} & p_3T + p_2p_3\dfrac{T^2}{2} + p_3p_5\dfrac{T^2}{2} \\ p_4T + p_4p_5\dfrac{T^2}{2} & p_4\dfrac{T^2}{2} & 1 + p_5T + p_5^2\dfrac{T^2}{2} \end{bmatrix} \tag{20}$$

$$\Psi = \begin{bmatrix} 0 \\ p_3 p_6 \dfrac{T^2}{2} \\ p_6 T + p_5 p_6 \dfrac{T^2}{2} \end{bmatrix} \tag{21}$$

By transferring the system equations from continuous to dicrete-time form, the time index will become discrete in the equation of the system output, so:

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -Jp_1 & 0 & -Jp_3 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} \tag{22}$$

From (22), the first and the third states can be written as a function of the outputs. On the other hand, the first and the second states has the relation $\dot{x}_1(t) = x_2(t)$. Using the forward rectangular approach [23], the following relations are acquired:

$$\begin{aligned} x_1(k) &= y_1(k) \\ x_2(k) &= \frac{y_1(k+1) - y_1(k)}{T} \\ x_3(k) &= -\frac{1}{Jp_3}(y_2(k) + Jp_1 y_1(k)) \end{aligned} \tag{23}$$

Using (23) in the discrete system model of (17), the following equation will be achieved:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \Phi \begin{bmatrix} y_1(k) \\ \dfrac{y_1(k+1) - y_1(k)}{T} \\ -\dfrac{1}{Jp_3}(y_2(k) + Jp_1 y_1(k)) \end{bmatrix} + \Psi[u_1(k)] \tag{24}$$

Use (22) at instant '$k+1$' and (24) to procure the following relations for outputs.

$$y_1(k+1) = x_1(k+1) = \left(\left(\phi_{11} - \frac{\phi_{12}}{T} + Jp_1 \frac{\phi_{13}}{-Jp_3}\right) y_1(k) - \frac{\phi_{13}}{Jp_3} y_2(k) + \psi_{11} u_1(k)\right) \left(1 - \frac{\phi_{12}}{T}\right)^{-1} \tag{25}$$

$$\begin{aligned} y_2(k+1) &= -Jp_1 x_1(k+1) - Jp_3 x_3(k+1) = \left(-Jp_1 - Jp_3 \frac{\phi_{32}}{T}\right) y_1(k+1) \\ &\quad + \left(-Jp_3\phi_{31} + Jp_3 \frac{\phi_{32}}{T} - Jp_3\phi_{33} \frac{Jp_1}{-Jp_3}\right) y_1(k) + \phi_{33} y_2(k) - Jp_3 \psi_{31} u_1(k) \end{aligned} \tag{26}$$

where $\phi_{11}, \phi_{12}, \ldots, \psi_{11}, \ldots$ are the components of Φ and Ψ in (20)–(21).

In (25), there is an inverse term post-multiplied by the whole equation. The inverse operation usually causes error in the mathematical computation. In this paper, the above statement is verified by several tests. From the simulation results, it seems reasonable to conclude that Eq. (25) magnifies computation error. To achieve a better result, we try to extract the system parameters from (26).

Using (20) and (26), one will reach the following equation:

$$\begin{aligned} y_2(k+1) &= (-Jp_1 - \tfrac{1}{2} Jp_3 p_4 T) y_1(k+1) + [\tfrac{1}{2} J(-p_3 p_4 p_5 + p_1 p_5^2) T^2 + \tfrac{1}{2} J(-p_3 p_4 \\ &\quad + 2p_1 p_5) T + Jp_1] y_1(k) + (1 + p_5 T + \tfrac{1}{2} p_5^2 T^2) y_2(k) + (-Jp_3 T(1 + \tfrac{1}{2} p_5 T) p_6) u_1(k) \end{aligned} \tag{27}$$

The coefficients of the signals y_1, y_2 and u_1 in (27) are the unknown constant parameters. Assuming the coeffients as $h_1, \ldots h_4$, (27) simplifies to the following form:

$$y_2(k+1) = h_1 y_1(k+1) + h_2 y_1(k) + h_3 y_2(k) + h_4 u_1(k) \tag{28}$$

Apply (28) repeatedly to form the following relation:

$$\underbrace{\begin{bmatrix} y_2(2) \\ \vdots \\ y_2(k+1) \\ \vdots \\ y_2(N) \end{bmatrix}}_{Y_2} = \underbrace{\begin{bmatrix} y_1(2) & y_1(1) & y_2(1) & u_1(1) \\ \vdots & \vdots & \vdots & \vdots \\ y_1(k+1) & y_1(k) & y_2(k) & u_1(k) \\ \vdots & \vdots & \vdots & \vdots \\ y_1(N) & y_1(N-1) & y_2(N-1) & u_1(N-1) \end{bmatrix}}_{U} \underbrace{\begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix}}_{H} \quad (29)$$

It is obvious that (29) is a regression equation. Definitely, it's solution is:

$$H = (U^{\mathrm{T}}U)^{-1}U^{\mathrm{T}}Y_2 \quad (30)$$

which gives the unknown parameters $h_1 \ldots h_4$. It is obvious from (27)–(28) that:

$$h_1 = -Jp_1 - \tfrac{1}{2}Jp_3p_4T \quad (31)$$

$$h_2 = \tfrac{1}{2}J(-p_3p_4p_5 + p_1p_5^2)T^2 + \tfrac{1}{2}J(-p_3p_4 + 2p_1p_5)T + Jp_1 \quad (32)$$

$$h_3 = 1 + p_5T + \tfrac{1}{2}p_5^2T^2 \quad (33)$$

$$h_4 = -Jp_3T(1 + \tfrac{1}{2}p_5T)p_6 \quad (34)$$

These four nonlinear equations with some other equations that will be derived in the succeeding sections, constitute a set of nonlinear equations. That set will be solved in Section 3.3.

3.2. Computing system eigenvalues using Prony method

The Prony method is a procedure for fitting one or more signals $y(t)$ to a weighted sum of exponential terms:

$$\hat{y}[t] = \sum_{i=1}^{n} B_i\,\mathrm{e}^{\lambda_i t} \quad (35)$$

The eigenvalues (λ_i) and the residue associated with each eigenvalue (B_i) are determined by fitting $\hat{y}(t)$, in a least-square sense, to the system output [24]. Observing the response of the system to an initial condition, a regression equation is solved in the Prony approach, to find the system eigenvalues.

The details of the Prony method can be found in refs. [25,26]. In this paper, the Prony method using multiple signals is used (see ref. [27]) to compute system eigenvalues more accurately.

If $n = 3$ is assumed as the system order in the Prony method, three eigenvalues α, β and γ are obtained. The characteristic equation of the system will become:

$$\Delta = (s-\alpha)(s-\beta)(s-\gamma) = s^3 - (\alpha+\beta+\gamma)s^2 + (\alpha\beta + \alpha\gamma + \beta\gamma)s - \alpha\beta\gamma \quad (36)$$

On the other hand, from (15) the characteristic equation will be:

$$\Delta = \det\left(\mathrm{sI} - \begin{bmatrix} 0 & 1 & 0 \\ p_1 & p_2 & p_3 \\ p_4 & 0 & p_5 \end{bmatrix}\right) = \det\left(\begin{bmatrix} s & -1 & 0 \\ -p_1 & s-p_2 & -p_3 \\ -p_4 & 0 & s-p_5 \end{bmatrix}\right)$$
$$= s^3 - (p_2+p_5)s^2 + (p_2p_5 - p_1)s + (p_1p_5 - p_3p_4) \quad (37)$$

Comparing (36) and (37), the following equations are procured.

$$(p_2 + p_5) = \alpha + \beta + \gamma \quad (38)$$

$$p_2p_5 - p_1 = \alpha\beta + \alpha\gamma + \beta\gamma \quad (39)$$

$$p_1p_5 - p_3p_4 = -\alpha\beta\gamma \quad (40)$$

These three equations and the four equations derived in Section 3.1 lead to a set of nonlinear equations which will be solved in the next sub-section.

3.3. Deriving parameters equations

In this subsection, the set of nonlinear Eqs. (31)–(34) and (38)–(40) are solved. Using (33):

$$p_5 = \frac{-1 \pm \sqrt{(-1+2h_3)}}{T} \tag{41}$$

The above formula results in two different values for p_5. Looking into (10), (13) and (15), it is obvious that $p_5 = -x_d/(x'_d T'_{do})$. Noting that x_d, x'_d and T'_{do} are positive, p_5 must satisfy the inequality $p_5 < 0$.

Moreover, from (38):

$$p_2 = -p_5 + (\alpha + \beta + \gamma) \tag{42}$$

p_2 must be negative too, because $p_2 = -D/J < 0$.

(41)–(42) result in two sets of responses for each unknown parameter. Among each of these two sets, the set which leads to negative numbers for both p_2 and p_5 would be acceptable. If both sets result in $p_5 < 0$ and $p_2 < 0$, the procedure discussed in this paper can be followed step by step for both sets. At last, each set of the parameters leads to a separate model of the form (15). The error between the model response and the measured signals should be calculated. Comparing the results, the parameter set which results in a model that its response is closer to the measured value, can be selected as the acceptable set.

Using (39) and (42) we have:

$$p_1 = -p_5^2 + (\alpha + \beta + \gamma)p_5 - (\alpha\beta + \alpha\gamma + \beta\gamma) \tag{43}$$

Looking deeply into the set of nonlinear Eqs. (31)–(34) and (38)–(40), it can be seen that for the three parameters p_3, p_4 and p_6, only two terms p_3p_4 and p_3p_6 are present in the equations. If the terms are defined as τ and θ respectively, it is seen that the parameters are decreased to six unknowns. In other words, from the above set of equations, not all the unknowns, but only six, can be found.

In order to have a logical set of equations, two equations are mixed together. According to (31), $J(p_1 + 1/2p_3p_4T) = -h_1$. Replacing this relation and (40) in (32), we will acquire the following formula for calculating J:

$$J = \frac{h_1 + h_2}{-(1/2)p_5(\alpha\beta\gamma)T^2 - (\alpha\beta\gamma)T}, \quad p_5 \neq -\frac{2}{T} \tag{44}$$

Using (34), it is obvious that:

$$p_3 = \frac{h_4}{-JT(1 + (1/2)p_5T)p_6} = \frac{\theta}{p_6}, \quad p_5 \neq -\frac{2}{T} \tag{45}$$

Using (40) and (45), p_4 is derived as below:

$$p_4 = \frac{(-p_5^3 + (\alpha+\beta+\gamma)p_5^2 - (\alpha\beta + \alpha\gamma + \beta\gamma)p_5 + \alpha\beta\gamma)p_6}{\theta} = \eta p_6 \tag{46}$$

It is seen that for each unknown parameter, a separate formula is derived. If the parameter 'p_6' is calculated, all the other parameters can be estimated from the corresponding formulas.

3.4. Steady state analysis

Assume that the measurement instruments for measuring the steady state value of the active and reactive load power (P and Q) and the infinite-bus voltage (V) at steady state are installed at the desired locations. Due to the following relations, the steady state values of phase and current are calculated [18,19]:

$$\phi = \tan^{-1}\left(\frac{Q}{P}\right) \tag{47}$$

$$I = \frac{\sqrt{P^2 + Q^2}}{V} \tag{48}$$

From (2), noting that δ and e'_q are the first and the third system states, it is concluded that:

$$x_{3o} = x'_d i_{do} + V\cos(x_{1o}) \tag{49}$$

Comparing (10) and (15) and replacing p_3 from (45), we will come to $K_2 = -Jp_3 = -J(\theta/p_6)$. Considering this last equation and (12), the following result is achieved:

$$x'_{\mathrm{d}} = \frac{V\ \sin(x_{1\mathrm{o}})}{-J\theta} p_6 \tag{50}$$

On the other hand, $i_{\mathrm{d}} = I \sin(\delta + \phi)$ [18,19]. So:

$$x_{3\mathrm{o}} = \frac{V\ \sin(x_{1\mathrm{o}})}{-J\theta} I\ \sin(x_{1\mathrm{o}} + \phi) p_6 + V\ \cos(x_{1\mathrm{o}}) \tag{51}$$

Finally, replacing (50)–(51) into (11), yeilds:

$$K_1 = \frac{-J\theta}{\sin(x_{1\mathrm{o}}) p_6} \left(\frac{V\ \sin(x_{1\mathrm{o}})}{-J\theta} I\ \sin(x_{1\mathrm{o}} + \phi) p_6 + V\ \cos(x_{1\mathrm{o}}) \right) \cos x_{1\mathrm{o}} + V^2 \left(\frac{1}{x_{\mathrm{q}}} - \frac{1}{x'_{\mathrm{d}}} \right) \cos(2x_{1\mathrm{o}}) \tag{52}$$

or

$$K_1 - VI\ \sin(x_{1\mathrm{o}} + \phi)\ \cos x_{1\mathrm{o}} = \begin{bmatrix} \dfrac{-JV(\cos(x_{1\mathrm{o}}))^2\theta}{\sin(x_{1\mathrm{o}})} & V^2\ \cos(2x_{1\mathrm{o}}) \end{bmatrix} \begin{bmatrix} \dfrac{1}{p_6} \\ \left(\dfrac{1}{x_{\mathrm{q}}} - \dfrac{1}{x'_{\mathrm{d}}} \right) \end{bmatrix} \tag{53}$$

In the above equation, some of the parameters such as V and $x_{1\mathrm{o}}$ are assumed to be measured. I and ϕ are calculated from (47)–(48). J, θ and K_1 are computed by the procedure given in the previous sub-section. Only the vector $[\,1/p_6 \quad ((1/x_{\mathrm{q}}) - (1/x'_{\mathrm{d}}))\,]$ is unknown. Testing the above formula for different operating points, these two unknown parameters can be calculated, too. Note that by neglecting the influence of magnetic saturation, x'_{d} and x_{q} will be constant for all operating conditions.

For two different operating points o_1 and o_2, (53) is extended to the following regression equation:

$$\begin{bmatrix} K_1 - VI\ \sin(x_{1\mathrm{o}} + \phi)\ \cos x_{1\mathrm{o}} \\ K'_1 - V'I'\ \sin(x_{1\mathrm{o}'} + \phi')\ \cos x_{1\mathrm{o}'} \end{bmatrix} = \begin{bmatrix} \dfrac{-JV(\cos(x_{1\mathrm{o}}))^2\theta}{\sin(x_{1\mathrm{o}})} & V^2\ \cos(2x_{1\mathrm{o}}) \\ \dfrac{-J'V'(\cos(x_{1\mathrm{o}'}))^2\theta'}{\sin(x_{1\mathrm{o}'})} & V'^2\ \cos(2x_{1\mathrm{o}'}) \end{bmatrix} \begin{bmatrix} \dfrac{1}{p_6} \\ \dfrac{1}{x_{\mathrm{q}}} - \dfrac{1}{x'_{\mathrm{d}}} \end{bmatrix} \tag{54}$$

Solving the above equation, all the unknown parameters of nonlinear model (16), could be computed. It is obvious that not only the parameters of the nonlinear model are identified, but also all the parameters of the synchronous generator can be extracted from that model.

The suggested identification method of this paper is summarized below:

(a) Select a proper input signal to be applied to the system. The input signal should have a wide spectrum of frequency to cover all system dynamics. It should also have a proper magnitude. The magnitude should be large enough to overcome the system noise and also should be small enough to be safe to perform the test.
(b) Select a proper sampling time and final time (the total time for the experiment).
(c) Apply the selected input signal (item (a)), to the system and sample the input–output data by a data acquisition system.
(d) If the measurements are noisy, use a suitable filter to filter out the noise from measurements.
(e) According to the filtered input–output data, find H from (30).
(f) Disconnect the input from the system ($u(t) = 0$) and find the unforced output to an initial condition. Sample the output data by a data acquisition system. If the measurements are noisy, use a suitable filter to filter out the noise from measurements.
(g) Use the Prony method to find system eigenvalues (α, β, γ). It is better to omit the bias from the signals and then find the system eigenvalues.
(h) Use (41)–(46) to find unknown variables: J, p_1, p_2, p_5, θ, η.
(i) Observe the steady state values in some different initial conditions. Solve (54) in order to find the parameters p_6 and $((1/x_{\mathrm{q}}) - (1/x'_{\mathrm{d}}))$.
(j) With the estimated parameters, obtain the model in (6). Find the model output ($\hat{y}_k$) and compare with the measured values.

To overcome the noise effect and reach more accurate results, it is better to do the test for different operating points and derive different models for the same system. The average value of the parameters derived in various tests, can be a better estimate of parameters.

4. Simulation results

The identification method, described in Section 3 is used to identify the third order model of a synchronous generator.

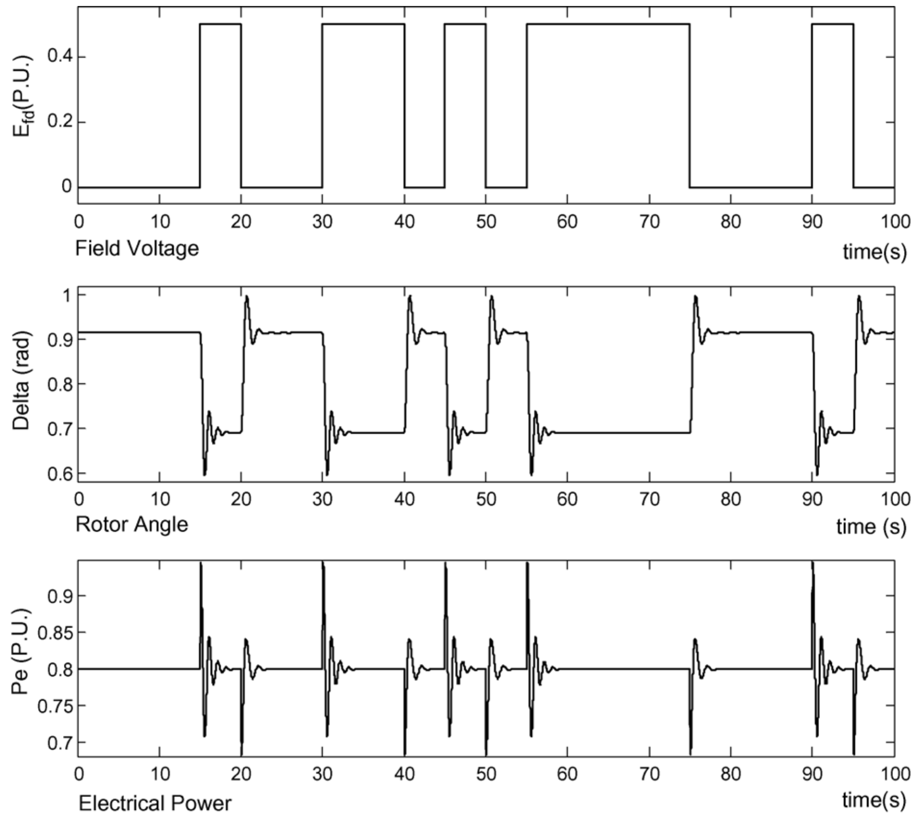

Fig. 2. Data collected from the third order synchronous generator nonlinear model.

The initial condition parameters used for system simulation are $\delta_0 = 0.9145$ rad and $e'_{q0} = 0.9653$ pu.

To illustrate the proposed identification method, a pseudo-random binary sequence (PRBS) signal is applied to the field voltage. The electric power, the rotor angle and the field voltage are sampled. The sampling time was selected to be 5 ms. This sampling time is fast enough to capture all system dynamics.

The input/output data collected from the system model, shown in Fig. 2, is used for the identification procedure.

The first step towards the identification algorithm is to solve the regression equation in (29) using the collected measured data. Since in this step, the theoretical relations of the algorithm are based on the linearized model of the synchronous generator, it is advised to omit the bias value of the measured data and capture the transient signal. Using the real measured data, without omiting the bias, may cause some error in parameter estimation.

Considering the above hint and omiting the bias in the measured signals, the H vector in (30) is calculated as follows:

$$H = [\,1.2329 \quad -1.1629 \quad 0.8588 \quad 0.0484\,]$$

Assuming the sampling time of 5 ms and the total time of 100 s for the experiment, the matrix U in (29) will have the size of $20{,}000 \times 4$.

At the second step, the input to the system is disconnected and the unforced system output to an initial condition is recorded.

Since the Prony method is completely separated from the previous step of the algorithm, it is not necessary to measure the same signals as the previous part. To achieve more accuracy, more signals can be measured. Here, the terminal voltage of the generator is measured, too. The sample signals of electrical power and terminal voltege are shown in Fig. 3.

The eigenvalues calculated by the Prony method are $\{-1.5556+4.2428i,\ -1.5556-4.2428i,\ -29.3131\}$, so the right terms in (38)–(40) becomes:

$$\alpha + \beta + \gamma = -32.4243$$
$$\alpha\beta + \alpha\gamma + \beta\gamma = 111.6179$$
$$\alpha\beta\gamma = -598.5979$$

(41)–(42) are used to calculate the unknown parameters. The following two sets of answers for p_2 and p_5 are calculated:

$$p_5 = \{-30.5790 \quad -369.4210\}$$
$$p_2 = \{-1.8453 \quad 336.9967\}$$

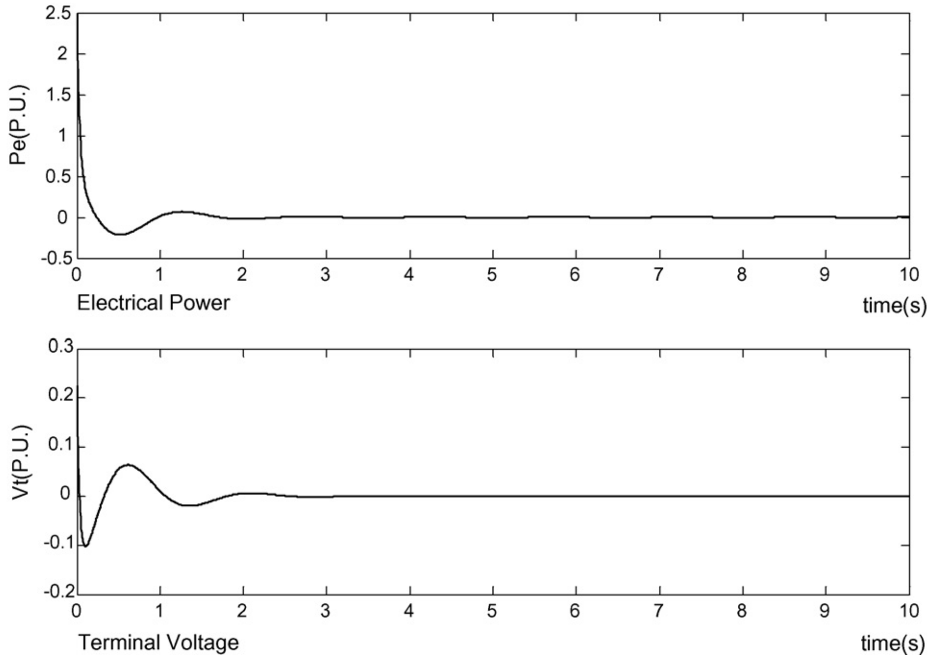

Fig. 3. Output data collected from the third order synchronous generator model to an initial condition.

It is mentioned in Section 3.3 that both p_2 and p_5 must be negative. The first set of the above results, satisfies this condition, but the second set does not. Omitting the second set and continuing with (43)–(46) the following results are achieved for the unknown variables:

$$p_5 = -30.5790;\quad p_2 = -1.8453;\quad p_1 = -55.1891;\quad J = 0.0253;\quad p_3 = \frac{-414.0395}{p_6};\quad p_4 = -2.4302p_6$$

At the third step, the steady state analysis is done. Measuring steady state parameters in two different operating conditions ($P = 0.8$ pu; $Q = 0$; $V = 0.98$ pu and $P = 0.95$ pu; $Q = 0.2$ pu; $V = 1.00$ pu), the last two unknowns are calculated:

$$p_6 = 7.60,\qquad \left(\frac{1}{x_{\mathrm{q}}} - \frac{1}{x'_{\mathrm{d}}}\right) = -1.1420$$

Finally, a comparison of the simulated model data and the identified model data, for the state space models in (15)–(16), is given in Table 1.

From the table it is seen that some parameters such as J or p_5 which are related to the synchronous generator parameters are estimated very well. A little error is seen in the estimation of parameters such as p_1 and p_4 which are related to the operating point condition.

The coresponding synchronous generator parameters are extracted from the above parameters and compared with the simulated model (reference model) parameters in Table 2. Note that the reactances are the augmented reactances, i.e. the line and transformer reactances should be substracted from them.

If the proposed technique is applied to a real system, it is obvious that we do not have the actual value of the parameters, so when the model is derived by the method of this paper, we can just compare the model response with the measured signals to evaluate the model. The procedure is the same as the following sub-section.

Table 1
State space system model parameters

	Simulated model data	Identified model data
p_1	−51.2198	−55.1891
p_2	−1.9842	−1.8453
p_3	−54.2447	−54.4788
p_4	−17.7075	−18.46952
p_5	−30.4401	−30.579
p_6	7.6336	7.6
J	0.0252	0.0253
$((1/x_{\mathrm{q}}) - (1/x'_{\mathrm{d}}))$	−1.1187	−1.1420

Table 2
Synchronous generator parameters

	Simulated model data	Identified model data
J (pu)	0.0252	0.0253
D (pu)	0.05	0.047
x'_d (pu)	0.568	0.563
x_d (pu)	2.072	2.265
x_q (pu)	1.559	1.576
T'_{do} (s)	0.1310	0.13158

4.1. Model validation

To evaluate the obtained nonlinear state space model, the system is excited by different inputs in the field voltage in different operating conditions and the performance of the system is observed. First, the operating point of $P = 0.8$ pu; $Q = 0$; $V = 0.98$ pu is selected. The system is excited by a pulse input with period 2 s and the magnitude of 1 pu. The electric power and the rotor angle are measured. The model is perturbed by the same input at the same operating point. The results are compared in Fig. 4.

To show that the identified model has successfully covered the main nonlinearities of the system, more studies were carried out. The performance of the identified model and the system at $P = 1.1$ pu; $Q = 0.4$ pu; $V = 1.02$ pu when the system is excited by a step increase of 0.5 pu in field voltage is compared in Fig. 5.

From Figs. 4 and 5 it can be seen that the proposed method is very successful in identifying the system dynamics. Additional studies were carried out to evaluate the obtained model when there is a measurement noise. The output measured signals are added with a color noise and the model is updated. The steady state analysis is the same as the previous test, but there is a bit change in transient response. The new nonlinear model parameter estimation are as follows:

$$P_5 = -29.0524; \qquad P_2 = -1.7375; \qquad J = 0.0259$$

The comparison of the performance of the identified model and the system at $P = 1.1$ pu; $Q = 0.4$ pu; $V = 1.02$ pu when the system is excited by a step change of 0.5 pu in field voltage is compared in Fig. 6.

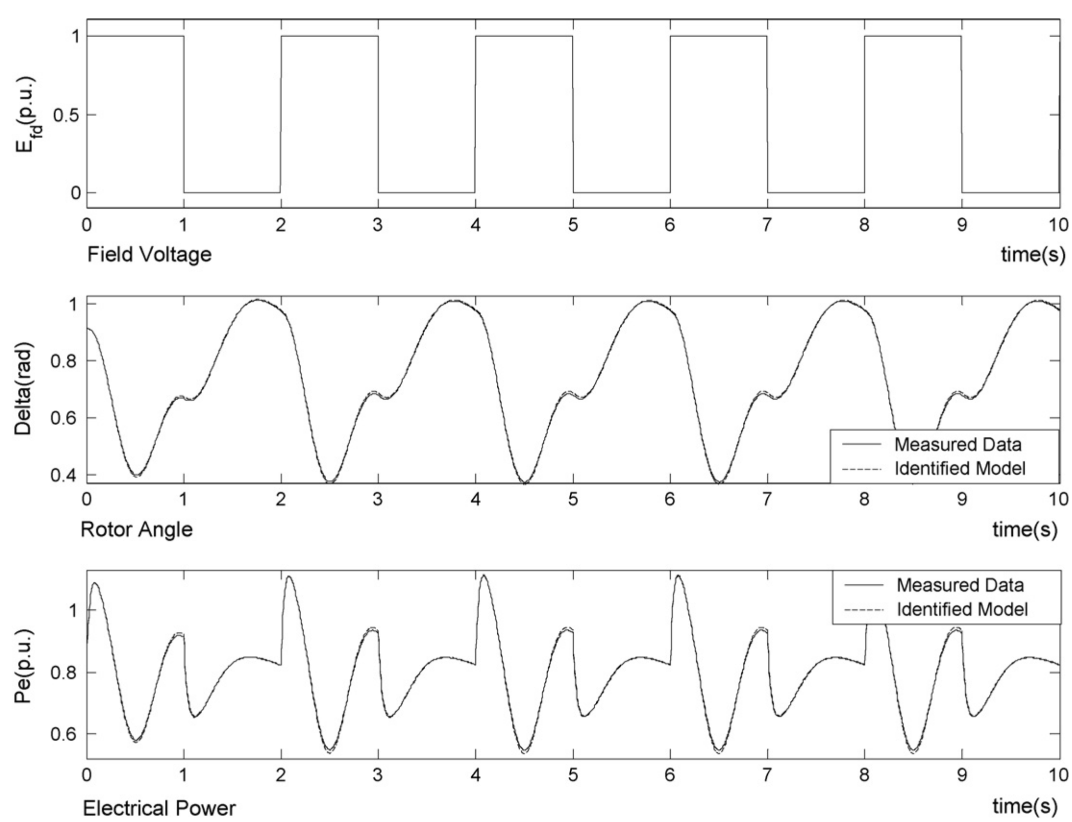

Fig. 4. The pulse input signal and the identification results at $P = 0.8$ pu; $Q = 0$; $V = 0.98$ pu.

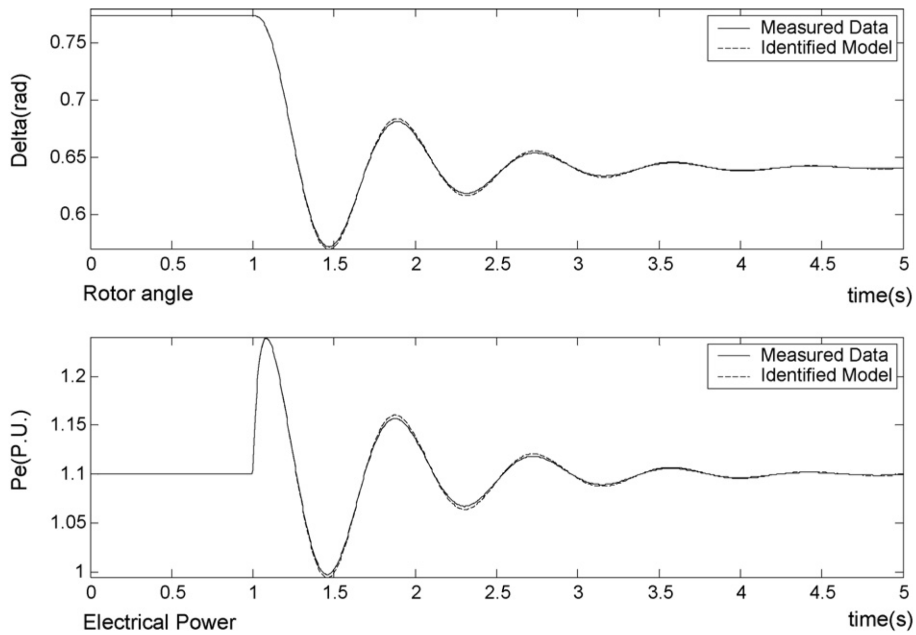

Fig. 5. Identification results at $P = 1.1$ pu; $Q = 0.4$ pu; $V = 1.02$ pu when the input is a step signal.

It is obvious that the measurement noise affects the parameter estimation. In Fig. 6, the error in estimation of the rotor inertia and damping factor causes the identified model response to have a delay relative to the measured data.

At the last step, the same PRBS signal shown in Fig. 2, is applied to the input of the system. The comparison of the measured data and the model output when no noise is present in the outputs and when there is a color noise in both outputs is shown in Figs. 7 and 8, respectively.

Comparing Figs. 7 and 8, it is obvious that the measurement noise affects the parameter estimation. In Fig. 7, the identified model and the measured signals are approximately the same. In Fig. 8, the identified model tracked the measured signal, but there is some error in the transients, specially the peak of signal is affected by noise. Furthermore, the error in estimation of the rotor inertia and damping factor caused the identified model response to have a delay relative to the measured data. However, the model could track the measured signal to some extent.

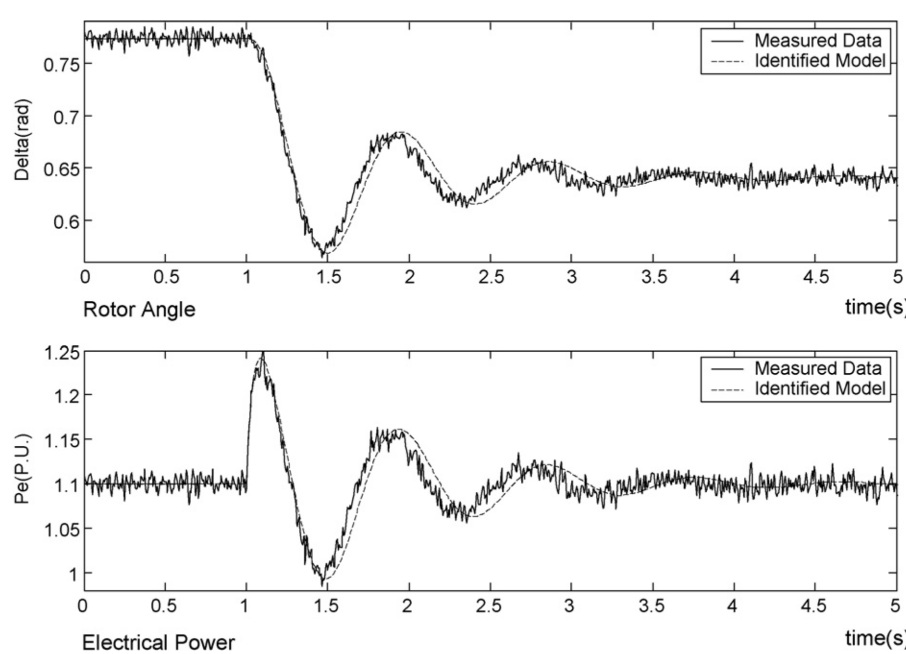

Fig. 6. Identification results at $P = 1.1$ pu; $Q = 0.4$ pu; $V = 1.02$ pu when the input is a step signal and the measurements are noisy.

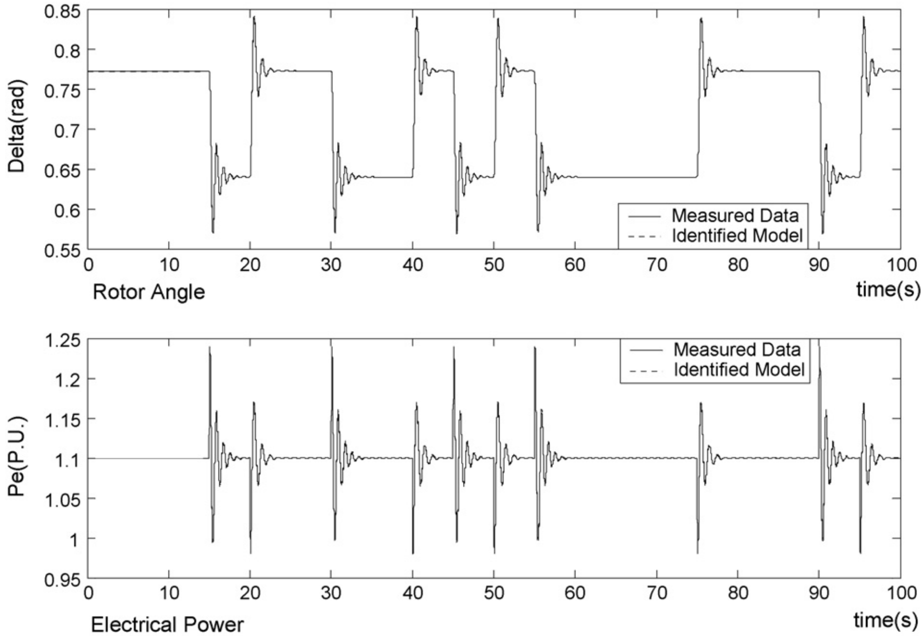

Fig. 7. Identification results at $P = 1.1$ pu; $Q = 0.4$ pu; $V = 1.02$ pu when the input is a PRBS signal.

In case of noisy measurements, it is more suitable to use some kind of filter in order to reduce the effect of noise. A third order low-pass filter with the cut-off frequency of 0.05 is used to filter the noise from the signal given in Fig. 8. The response of the model generated by the filtered signals is compared with the measured signals in Fig. 9.

Figs. 4–9 show that the identified model has modeled the system correctly. When there is no noise in the measurements, the transient response of the nonlinear model and the simulated system are approximately the same and the method is very successful in identifying the model. When noise is present in the measured data, the identified model has some error and it is advised to filter out the noise before doing any process.

Simulation tests admits that the proposed method can be used successfully for identification of a synchronous machine model.

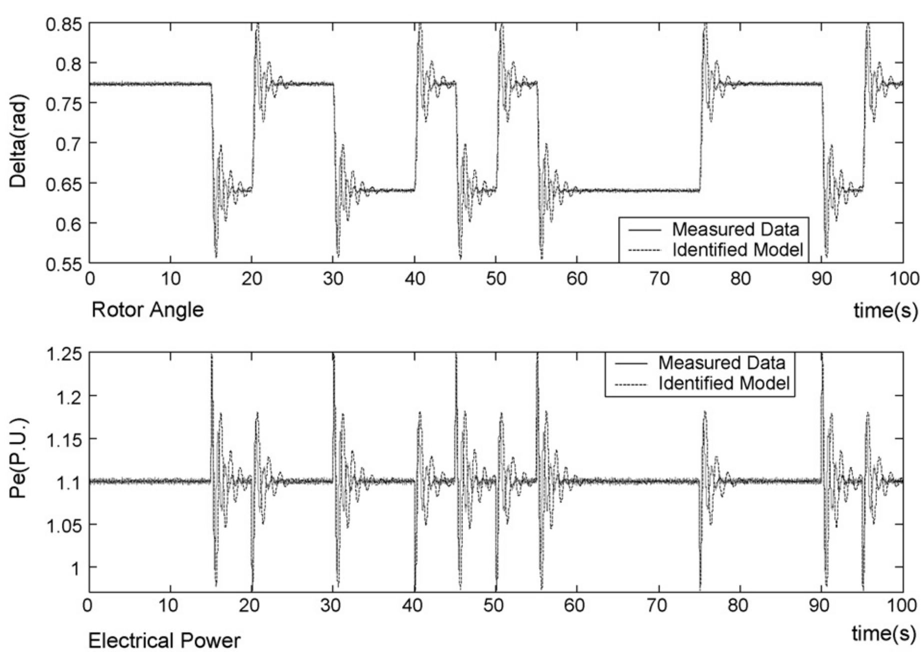

Fig. 8. Identification results at $P = 1.1$ pu; $Q = 0.4$ pu; $V = 1.02$ pu when the input is a PRBS signal and the measurements are noisy.

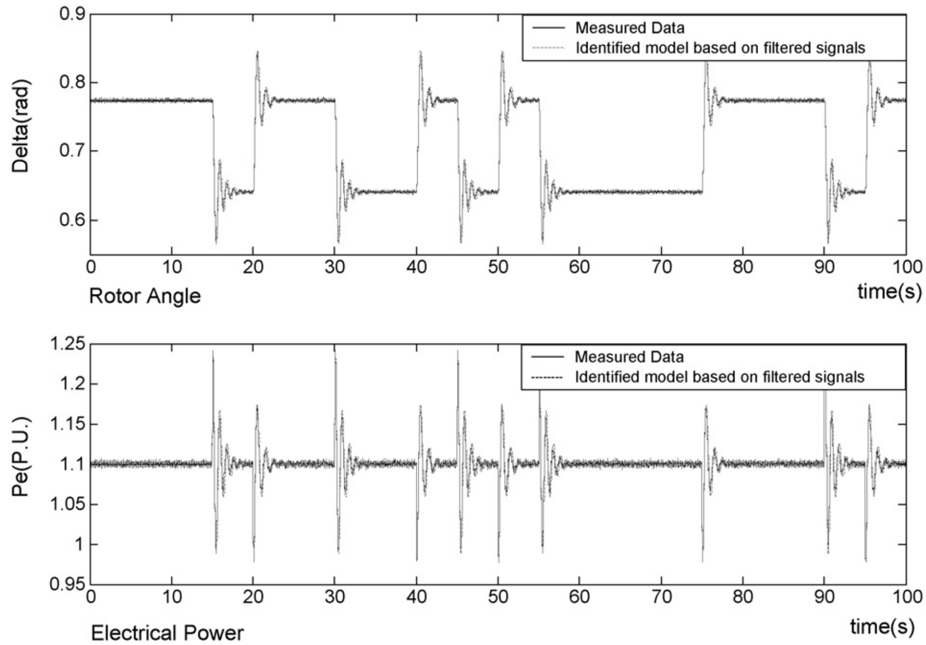

Fig. 9. Identification results when the signals in Fig. 8 are filtered.

5. Conclusions

Identification of a synchronous generator using a theoretical method is described in this paper. The proposed method is classified as grey-box modeling. The method uses the known Prony method and the theoretical relations of machine parameters to find the state space model of the system. In this paper, generator rotor angle and the active output power are considered as the outputs of the system and the field voltage as the input of the system.

Simulation results show that when the measurements are not noisy, the transient response of the nonlinear model and the simulated system are approximately the same. When noise is present in the measured data, some error is seen and it is advised to filter out the noise before doing any process. Simulation tests admit that the proposed method can be used successfully for identification of a synchronous machine model. The obtained grey-box model can be used for system analysis and controller design.

The method is useful in order to find the third-order model of a synchronous generator. Future works will involve extending the method to include the seventh-order model of a synchronous generator or to assume a synchronous generator as a part of a multi-machine power system.

Appendix A. List of symbols

The main variables of model (1) are:

e'_{q}	transient internal voltage of armature
E_{FD}	the equivalent EMF in the excitation coil
$i_{\mathrm{d}}, i_{\mathrm{q}}$	direct and quadrature axis stator currents
J, D	rotor inertia and damping factor
P, Q	terminal active and reactive power per phase
T'_{do}	direct axis transient time constant
T_{e}	output electric torque
T_{m}	input mechanical torque
v_{t}	generator terminal voltage
V	infinite bus voltage
x_{d}	direct axis reactance
x'_{d}	direct axis transient reactance
x_{q}	quadrature axis reactance

Greek letters

δ	rotor angle with respect to the machine terminals
ω	rotor speed

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