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Asymmetric Barrier Lyapunov Function Self Optimizing Control For Brushless DC Motor With Globalized Constrained Nelder-Mead Algorithm

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Abstract: This paper presents the design of a backstepping controller with dynamic surface control (DSC) based on an asymmetric barrier Lyapunov function (ABLF) for the physical constrained position control of a brushless DC (BLDC) motor. An integer-order friction observer is attached to the backstepping control law to estimate and compensate for the torque disturbance in the motor. A self optimizing control (SOC) layer based on the globalized constrained Nelder-Mead optimization (GCNM) algorithm is added on top of the backstepping controller. Thus, the overall controller can be made aware of the closed-loop system performance and can self-adjust the backstepping controller terms to ensure the system operation under certain desired specifications. Obtained results show that the proposed control method combined with an SOC layer ensures the stability and performance of the closed-loop system for long operation periods.

Keywords: Asymmetric Barrier Lyapunov Function, Backstepping Control, Globalized Constrained Nelder Mead, Dynamic Surface Control, Friction Compensation.

1. INTRODUCTION

Brushless DC (BLDC) motors are widely used in industry because of their high efficiency, low cost, and simple structure compared to brushed DC motors. One of the main challenges of BLDC motors is their controller complexity due to the inherent non-linear characteristics of the system. Also, non-linear friction is hard to estimate accurately, posing an additional challenge to the already complex non-linear controller.

There have been extensive studies on non-linear friction compensation like the classical approach of (Friedland and Park, 1992) where friction is modeled by multiplying a constant to the sign of velocity, which is used to estimate and cancel out the friction torque disturbances. Likewise, other adaptive friction compensation approaches like the fractional friction compensation (Qiu et al., 2015) may prove useful as well as (Friedland and Mentzelopoulou, 1992; Liao and Chien, 2000).

On the other hand, one of the challenges of the DC motor controller design is the inherent physical constraint of the operation space and capacity of the controller. Therefore is a growing interest in the control community to deal with the constraint problem. Methods like set invariance (Hu et al., 2002), reference governor (Gilbert and Kolmanovsky, 2002) and model predictive control (Mayne, 2014) are quiet sufficient for stability problems but not sufficient for safety challenges.

In the past decade, with the proliferation of control Lyapunov function (CLF) and control barrier Lyapunov function (CBLF) for vehicle safety control, the barrier Lyapunov function (BLF) and its variants such as asymmetric barrier Lyapunov function (ABLF) have been used to deal with the state, and output constraints for nonlinear systems in the Brunovsky form (Ngo et al., 2005). Extensive studies in (Tee et al., 2009; Ren et al., 2010), prove that BLF effectively deals with both symmetric and asymmetric output constraints for known or unknown nonlinear systems. These studies employed the backstepping control strategy.

Although the backstepping control is effective, the explosion of complexity arises as the system order n increases caused by the repeated differentiation inherent in each backstepping procedure (Krstic et al., 1995). This complexity presents unwanted complications and slows down the system convergence in real-time. To address this challenge, Dynamic Surface Control (DSC) (Swaroop et al., 2000) was developed, where a first-order filter is used to express the virtual input at each step of the backstepping procedure without considering constraints (Wang and Huang, 2005).

However, for the brushless DC motor, the long-time stable operation of the system is affected by the changes in the motor's physical constraint. For this reason, a supervisory control layer is required to monitor the closed-loop performance of the backstepping controller to keep the system's operating performance and stability under certain desired conditions.

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In this paper, a backstepping controller with dynamic surface control (DSC) based on an asymmetric barrier Lyapunov function (ABLF) is designed for the position control of a physically constrained brushless DC (BLDC) motor. An integer-order friction observer is attached to the backstepping control law to estimate and compensate for the torque disturbance in the motor. A self-optimizing control (SOC) layer uses the globalized constrained Nelder-Mead optimization (GCNM) algorithm proposed by (Viola and Chen, 2020) to improve the system performance in a long time batch sequential execution. It is performed by evaluating an economical cost function that quantifies the closed-loop performance of the brushless DC motor and the backstepping controller with ABLF in terms of its steady-state error.

The main contribution of this paper is the combination - for the first time - of backstepping DSC control based on ABLF functions with a torque friction observer for BLDC motors as in (Qiu et al., 2015) and a self-optimizing control layer that uses the GCNM optimization algorithm to improve the closed-loop performance of the non-linear brushless system.

2. DC MOTOR DYNAMIC MODEL AND THE FRICTION MODEL

The dynamic model of the brushless DC motor is given by (1), where θ and w_n are the motor angular position and velocity, i_m is the armature current, U is the applied input voltage, $E = K_e\omega_m$ is the back-EMF voltage, T_e is the motor electromagnetic torque, J is the moment of inertia, R and L are the motor resistance and inductance, K_e the voltage constant, K_t the motor torque constraint, and T_L the friction torque, which is unknown in most position tracking control systems, affecting the tracking performance. Thus, a friction torque observer is needed for accurate estimation and compensation for the BLDC tracking control. From (Friedland and Park, 1992), (2) is used to express the friction torque, where a is a parameter to be estimated by the observer.

$$\begin{aligned} \dot{\theta} &= \omega_m \\ J\dot{\omega}_m &= T_e - T_L = K_t i_m - T_L \\ U &= L i_m + R i_m + E \end{aligned} \tag{1}$$

$$T_L(\omega_m, a) = a_{\mathrm{L}}\mathtt{sgn}(\omega_m). \tag{2}$$

3. BARRIER LYAPUNOV FUNCTION

The following Definition, Assumptions and Lemmas from (Qiu et al., 2015) are used to define and establish the constraints satisfaction and performance bounds for the barrier Lyapunov function.

Definition 3. *Barrier Lyapunov function is a continuously differentiable and positive definite scalar function $V(x)$, defined with respect to the system $\dot{x} = f(x)$ on an open region D containing the origin. It has continuous first-order partial derivatives at every point of D. It has the property $V(x) \to \infty$ as x approaches the boundary of D, and satisfies $V(x(t)) \le b$, $\forall t \ge 0$ along the solution of $\dot{x} = f(x)$ for $x(0) \in D$ and some positive constants b.*

Assumption 1. *There exist constants $\underline{K}_{ci}$ and $\overline{K}_{ci}$, $i = 0, 1, 2$ satisfying $\underline{k}_{c1}(t) \ge \underline{K}_{c0}$ and $\overline{k}_{c1}(t) \le \overline{K}_{c0}$ and their time derivatives satisfy $|\underline{k}_{ci}^{(i)}(t)| \ge \underline{K}_{ci}$, $|\overline{k}_{ci}^{(i)}(t)| \le \overline{K}_{ci}$, $i = 1, 2, \forall t \ge 0$.*

Assumption 2. *There exist functions $\underline{Y}_0$, $\overline{Y}_0$, and positive constants Y_1, Y_2 satisfying $\underline{Y}_0(t) > \underline{k}_{c1}$ and $\overline{Y}_0(t) < \overline{k}_{c1}(t)$ such that the desired trajectory $y_d(t)$ and its derivative satisfy $Y_0(t) \le y_d(t) \le \overline{Y}_0(t)$ and $|\dot{y}_d(t)| \le Y_1$, $|\ddot{y}_d(t)| \le Y_2$, $\forall t \ge 0$, implying that they are continuous and available in a compact set $\Omega_{yd} := \{y_d \in \mathbb{R} : y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \le \delta_{yd}\} \subset \mathbb{R}$.*

Assumption 3. *The function g_i is positive and there exists a class of positive constants g_{imin} and g_{imax} such that $0 < g_{imin} \le g_i(\overline{x}_i) \le g_{imax}$ for $y = x_1$ satisfying $\underline{k}_{c1}(t) < y(t) < \overline{k}_{c1}(t)$, $\forall t \ge 0$.*

Lemma 1.(Ren et al., 2010).*For any functions $k_{a1}(t)$, $k_{b1}(t)$, let $\mathbf{S_1} := \{S_1 \in \mathbb{R} : -k_{a1}(t) < S_1 < k_{b1}(t)\} \subset \mathbb{R}$ and $N := \mathbb{R}^l \times \mathbf{S_1} \subset \mathbb{R}^{l+1}$ be open sets. Consider the system*

$$\dot{\eta} = h(t, \eta)$$

where $\eta := [\omega, S_1]^T \in N$, and $h : \mathbb{R}_+ \times N \to \mathbb{R}^{l+1}$ is piecewise continuous with respect to η, uniformly with respect to t, on $\mathbb{R}_+ \times N$. Suppose that there exist functions $U : \mathbb{R}^l \to \mathbb{R}_+$ and $V_1 : \mathbf{S_1} \to \mathbb{R}_+$ continuously differentiable and positive definite in their respective domains, such that

$$\begin{aligned} V_1(z) \to \infty \text{ as } z \to -k_{a1}(t) \quad \text{or} \quad z \to k_{b1}(t) \\ \gamma_1(||\omega||) \le U(\omega) \le \gamma_2(||\omega||) \end{aligned}$$

where γ_1 and γ_2 are class K_∞ functions. Let $V(\eta) := V_1(S_1) + U(\omega)$ and $S_1(0) \in \mathbf{S_1}$. If the following inequality holds:

$$\dot{V} = \frac{\partial V}{\partial \eta} h \le -cV + v$$

in the set $\eta \in N$, and c, v are positive constants, then $S_1(t)$ remains in the open set $\mathbf{S_1}$, $\forall t \in [0, \infty)$.

Lemma 2.(Ren et al., 2010).*For all $S_1 < k_{b1}$, the inequality $\log \frac{k_{b1}^2(t)}{k_{b1}^2(t) - S_1^2} \le \frac{S_1^2}{k_{b1}^2(t) - S_1^2}$ holds.*

4. BRUSHLESS MOTOR FRICTION TORQUE OBSERVER

According to (1), the dynamics of the brushless DC motor is represented by (3)

$$J\dot{\omega}_m = T_e - T_L = K_t i_m - T_L. \tag{3}$$

Using the nonlinear reduced-order observer proposed by Friedland and Park (1992), the estimator for a is given by (5), where $k > 0$ and $\mu > 0$ are design parameters and z is defined by,

$$\dot{z} = kJ|\omega_m|^{\mu-1}[K_t i - \hat{T}_L(\omega_m, a)]\mathrm{sgn}(\omega_m). \quad (4)$$

$$\hat{a} = z - kJ|\omega_m|^{\mu}. \quad (5)$$

Considering that (6) is the estimation error,

$$e = a - \hat{a}, \quad (6)$$

and assuming that the true parameter vector a is constant:

$$\begin{aligned} \dot{e} &= -\dot{\hat{a}} \\ &= -\dot{z} + kJ\mu|\omega_m|^{mu-1}J\dot{\omega}_m\mathrm{sgn}(\omega_m) \\ &= -k\mu|\omega_m|^{\mu-1}[T_L(\omega_m, a) - \hat{T}_L(\omega_m, a)]\mathrm{sgn}(\omega_m) \\ &= -k\mu|\omega_m|^{\mu-1}[(a - \hat{a})]\mathrm{sgn}(\omega_m) \\ &= -k\mu|\omega_m|^{\mu-1}e. \end{aligned} \quad (7)$$

Thus, from (7) it is proven that the observer error converges asymptotically to 0 if ω_m is bounded away from 0. So, the adaptive estimator of the load torque is given by,

$$\hat{T}_L = z - kJ|\omega_m|^{\mu}\mathrm{sgn}(\omega_m). \quad (8)$$

5. BACKSTEPPING DSC ABLF CONTROLLER DESIGN

In this section, the backstepping design procedure is applied step by step applying dynamic surface control using asymmetric barrier Lyapunov function for the brushless DC motor position tracking control. The control objective is to determine a continuous feedback control law such that the position output y tracks a desired position trajectory $y_d(t)$ while ensuring that all the closed-loop signals are bounded, and the corresponding constraints are not violated. Meanwhile, the friction effect is compensated in the second step of the backstepping design.

First the brushless DC motor model (1) is transformed into a general form with affine input;

$$\begin{aligned} \dot{x}_1 &= f_1(x_1) + g_1(x_1)x_2 \\ \dot{x}_2 &= f_2(x_2) + g_2(x_2)x_3 \\ \dot{x}_3 &= f_3(x_3) + g_3(x_3)u \\ y &= x_1, \end{aligned} \quad (9)$$

where

$$\begin{array}{ll} x_1 = \theta;\ x_2 = \omega_m; & x_3 = i_m;\ u = U \\ f_1 = 0; & g_1 = 1 \\ f_2 = -\dfrac{T_L}{J}; & g_2 = \dfrac{K_t}{J} \\ f_3 = -\dfrac{R}{L}i_m - \dfrac{K_e}{L}\omega_m; & g_3 = \dfrac{1}{L}. \end{array} \quad (10)$$

Step 1:

Assign $S_1 = y_d - x_1$ as the tracking error and $S_2 = x_2 - z_2$ as the virtual error. Then introduce a filtering virtual function z_2, and let α_1 pass through a first-order filter with a time constant τ_2 as follows,

$$\tau_2\dot{z}_2 + z_2 = \alpha_1, \qquad z_2(0) = \alpha_1(0), \quad (11)$$

where α_1 is a stabilizing function to be designed. From (11), express the output error of the first-order filter as $\chi_2 = z_2 - \alpha_1$ and $\dot{z}_2 = -\frac{\chi_2}{\tau_2}$. Choose an asymmetric barrier Lyapunov function candidate combined with quadratic Lyapunov function as follows,

$$V_1 = \frac{1 - q(S_1)}{2}\log\frac{k_{a1}^2}{k_{a1}^2 - S_1^2} + \frac{q(S_1)}{2}\log\frac{k_{b1}^2}{k_{b1}^2 - S_1^2} + \frac{1}{2}\chi_2^2 \quad (12)$$

in which,

$$q(S_1) = \begin{cases} 1, & 0 < S_1 \\ 0, & S_1 \leq 0 \end{cases}$$

$$k_{a1} = k_{c1} - Y_{l0},\ k_{b1} = k_{c1} - Y_{h0},$$

where $\log(\cdot)$ denotes natural logarithm, and k_{a1}, k_{b1} are the constraints on S_1. This is based on the perturbation of the upper and lower bounds of the desired trajectory y_d. It is clear that an asymmetric barrier Lyapunov function candidate could relax the requirement of initial conditions on starting values of the output and afford greater flexibility on output constraints. $q(S_1)$ is abbreviated as q to be a simpler notation throughout this paper.

By inspection, V_1 is positive definite because $V_1 = 0$ if and only if $S_1 = 0$ and $z_2(0) = \alpha_1(0)$ simultaneously for $-k_{a1} < S_1 < k_{b1}$. Also, V_1 is piecewise smooth and continuously differentiable within each of the two intervals $S_1 \in (-k_{a1}, 0]$ and $S_1 \in (0, k_{b1})$ in terms with the fact that $\lim_{S_1 \to 0^+} \frac{dV_1}{dS_1} = \lim_{S_1 \to 0^-} \frac{dV_1}{dS_1} = 0$. Therefore, V_1 is C^1 and it is a valid Lyapunov candidate. Furthermore it is true that the output would not be violated out of the interval $S_1 \in (-k_{a1}, 0)$ and $S_1 \in (0, k_{b1})$. Design stabilizing function α_1 as follows,

$$\alpha_1 = \frac{1}{g_1}\{-[(1-q)(k_{a1}^2 - S_1^2) + q(k_{b1}^2 - S_1^2)]k_1 S_1 - f_1 + \dot{y}_d\}. \quad (13)$$

Taking the time derivative of χ_2,

$$\begin{aligned} \dot{\chi}_2 &= \dot{z}_2 - \dot{\alpha}_1 \\ &= -\frac{\chi_2}{\tau_2} + \zeta_2(x_1, y_d, \dot{y}_d, \ddot{y}_d) \end{aligned} \quad (14)$$

where

$$\zeta_2(x_1, y_d, \dot{y}_d, \ddot{y}_d) = \frac{\partial\alpha_1}{\partial x_1}\dot{x}_1 + \frac{\partial\alpha_1}{\partial y_d}\dot{y}_d + \frac{\partial\alpha_1}{\partial\dot{y}_d}\ddot{y}_d \quad (15)$$

is a continuous function and has a maximum M_2 in a compact set $\Omega_{yd} \times \Omega_y$ under the Assumptions 1 and 2. According to Young's inequality, it follows,

$$\begin{aligned} &|\chi_2\zeta_2| \leq \frac{1}{2\sigma_2}\chi_2^2\zeta_2 + \frac{\sigma_2}{2} \leq \frac{1}{2\sigma_2}\chi_2^2 M_2 + \frac{\sigma_2}{2}, \sigma_2 > 0 \\ &g_1 S_1\chi_2 \leq g_{1max}\left(S_1^2 + \frac{\chi_2^2}{4}\right). \end{aligned} \quad (16)$$

The time derivative of V_1 can be calculated as:

$$\begin{aligned}\dot{V}_1 &= \left(\frac{1-q}{k_{a1}^2 - S_1^2} + \frac{q}{k_{b1}^2 - S_1^2}\right) S_1 \dot{S}_1 + \chi_2 \dot{\chi}_2 \\ &= -k_1 S_1^2 + \left(\frac{1-q}{k_{a1}^2 - S_1^2} + \frac{q}{k_{b1}^2 - S_1^2}\right)(g_1 S_1 S_2 + g_1 S_1 \chi_2) \\ &- \frac{\chi_2^2}{\tau_2} + \chi_2 \zeta_2.\end{aligned} \tag{17}$$

By substituting (16), the following inequality is attained:

$$\begin{aligned}\dot{V}_1 \leq &-\left[k_1 - g_{1max}\left(\frac{1-q}{k_{a1}^2 - S_1^2} + \frac{q}{k_{b1}^2 - S_1^2}\right)\right] S_1^2 \\ &- \left[\frac{1}{\tau_2} - \frac{g_{1max}}{4}\left(\frac{1-q}{k_{a1}^2 - S_1^2} + \frac{q}{k_{b1}^2 - S_1^2}\right) - \frac{M_2^2}{2\sigma_2}\right]\chi_2^2 \\ &+ \frac{\sigma_2}{2} + g_1 S_1 S_2 \left(\frac{1-q}{k_{a1}^2 - S_1^2} + \frac{q}{k_{b1}^2 - S_1^2}\right)\end{aligned} \tag{18}$$

Therefore, the selection range of constant gain k_1 and time constant τ_2 should be limited to $k_1 > g_{1max}\left(\frac{1-q}{k_{a1}^2 - S_1^2} + \frac{q}{k_{b1}^2 - S_1^2}\right)$ and $\frac{1}{\tau_2} \geq \left[\frac{g_{1max}}{4}\left(\frac{1-q}{k_{a1}^2 - S_1^2} + \frac{q}{k_{b1}^2 - S_1^2}\right) + \frac{M_2^2}{2\sigma_2}\right]$ in order to guarantee closed-loop stability. The last term $g_1 S_1 S_2\left(\frac{1-q}{k_{a1}^2 - S_1^2} + \frac{q}{k_{b1}^2 - S_1^2}\right)$ would be canceled in the subsequent steps.

The same procedure is repeated for step 2 (Qiu et al., 2015) which leads to the final step 3.

Step 3: From step 2, $S_3 = x_3 - z_3$. Choose a Lyapunov function candidate as:

$$V_3 = V_2 + \frac{1}{3} S_3^2. \tag{19}$$

The final control law becomes;

$$u = \frac{1}{g_3}\left(- k_3 S_3 - f_3 - g_2 S_2 - \frac{\chi_3}{\tau_3}\right) \tag{20}$$

The complete proof can be confirmed in (Qiu et al., 2015).

6. SELF OPTIMIZING CONTROL LAYER FOR THE BACKSTEPPING ABLF CONTROLLER

In this section, an online self optimizing control (SOC) layer is added to the backstepping ABLF controller to improve the steady state performance of the brushless DC motor system for a longer operation period as shown in Fig. 1. As can be observed, the SOC control layer is added on top of the system, which monitors the closed-loop performance under a repetitive task defined as a square reference signal r looking for the optimization of the controller parameters k_1, k_2, and k_3 that minimize the economic cost function (21) defined as the position steady state error e_{ss} of the motor.

$$J = e_{ss} \tag{21}$$

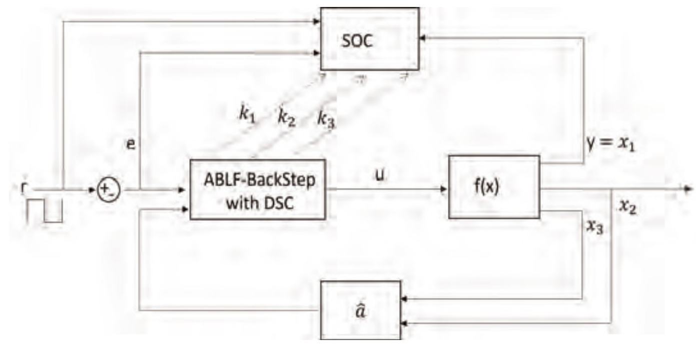

Fig. 1. Self-Optimizing Control Architecture for Brushless DC motor system

In this paper, the Globalized Constrained Nelder-Mead (GCNM) optimization algorithm proposed by Viola and Chen (2020) is employed as optimization algorithm on the SOC control layer. It is an adaptation of the classic Nelder-Mead algorithm (Nelder and Mead, 1965) designed to operate online with the closed-loop system. The flowchart for the GCNM algorithm is illustrated in (Viola and Chen, 2020). As can be observed, the GCNM method employs the same operations performed by the original NM algorithm: evaluation, reflection, contraction, expansion, and shirking, to create the simplex shape consisting of $n+1$ vertices where n is the number of parameters of the optimization. The operations of reflection, contraction, expansion in the GCNM algorithm are associated with a constant α, β, γ, δ respectively which are selected as $\alpha = 1, \beta = 1 + 2nr, \gamma = 0.75 - 0.5nr, \delta = 1 - nr$ with $nr = 1/n$ as the reciprocal of the number of dimensions following the adaptive rule for NM algorithm proposed by (Gao and Han, 2012).

On the other hand, considering that the classic NM algorithm is prone to fall into local minimum, the GCNM algorithm introduces a probabilistic restart mechanism that reset the search from a different random initial condition to prevent it from falling into a local minima. Initially, the probabilistic restart evaluates if the cost function has reached a steady value. For this, the algorithm evaluates if the standard deviation of the last m values of the simplex centroid is below a tolerance threshold ϵ. If this is true, then the optimization is in the steady state, and the constraints are evaluated. If at least one of the constraints is not satisfied, the GCNM restarts the search on a new random point assigning a new set of initial conditions among the parameter space defined for the problem following a gaussian sampling distribution.

One of the main advantages of the GCNM algorithm is that it can be used for any system without prior knowledge of its dynamical model as a derivative-free algorithm. Likewise, considering the sequential structure of the GCNM algorithm and the low computational complexity of the algorithm, it can be adapted for real-time execution, conditioning the computation of each of the operations to a period of the reference signal r. The GCNM have been implemented on a First Order Plus Dead Time (FOPDT) system where it successfully optimized the system performance. In this paper, for the first time, the GCNM is implemented on a non-linear system - the Brushless DC Motor.

7. OBTAINED RESULTS

A simulation benchmark was built in Simulink/Matlab for the backstepping ABLF controller with SOC control layer following the procedure presented on sections 5 and 6. The system was tested for a reference square wave signal of amplitude $\pi/2$ and frequency of 2Hz. The parameters of the brushless DC motor used for this simulation are $J = 1.8 \times 10^{-2}\ kg.m^2$, $R = 0.21\ \Omega$, $L = 0.003$ H, $K_e = 9.55 \times 10^{-3}\ V/rad.s^{-1}$, $K_t = 9.55 \times 10^{-3}\ Nm/A$. The files for the backstepping ABLF controller can be downloaded at `https://github.com/jnwok/ABLF_SOC_benchmark.git`.

The simulation results for the backstepping ABLF controller without the SOC control layer are shown in Fig 2 using as controller gains $k_1 = 10,\ k_2 = 1,\ k_3 = 10,\ \tau_1 = 0.05 \times 10^{-2},\ \tau_2 = 10^{-2}$. As can be observed for a time less than $t = 10s$, the controller starts tracking the reference signal and keeps it stable until $t = 40s$. Then at $t = 40s$, the system dynamics induces a tracking loss that is compensated by the backstepping ABLF controller after 10 seconds. It indicates that a supervisory layer that monitors the closed-loop performance of the system is required to avoid the tracking loss. For this reason the SOC control layer is introduced, using the controller gains for the backstepping ABLF controller as initial condition for the GCNM algorithm.

Figure 3 shows the performance of the backstepping ABLF controller with SOC control layer for the position tracking task. It can be seen that the GCNM optimization algorithm compensates not only for any unstable behavior from 40s to 50s, but also for the steady-state error of the motor position, making the system more robust against model disturbances and uncertainties. The convergence of the controller terms k_1, k_2 and k_3 are shown in Fig. 4. Once the SOC control layer is activated once the simulation starts, it finds the optimal values around $t = 10s$ corresponding to $k_1 = 2.931,\ k_2 = 0.474,\ k_3 = 20.693$, which minimizes the steady-state error. Likewise, Fig. 5 shows the evolution of the cost function which reaches its optimal value around $t = 10s$.

On the other hand, an offline optimization is performed to find the optimal values for $k_1,\ k_2,\ k_3$ employing the Simulink Design Optimization toolbox (SLDO) Mathworks Inc (2020) to compare its performance with the SOC online optimization, obtaining $k_1 = 6.272,\ k_2 = 2399,\ k_3 = 31.917$. The backstepping ABLF controller response with the gain values obtained from SLDO optimization is shown in Fig 6. It can be observed that the controller with the gains obtained with the SLDO compensates for the instability at $t = 40s$ shown in Fig. 2. However, it is not able to compensate for the steady-state error as when the SOC control layer is introduced. Likewise, there is no overshoot for the backstepping ABLF controller when the SLDO optimization is performed, but its settling time is bigger compared to the controller with and without SOC layer. In addition, the SLDO finds a very high gain for $k_2 = 2399$, which could lead to high noise amplification in a real implementation.

Likewise, a performance comparison between the backstepping ABLF controllers is shown in Table 1. As can be observed, the backstepping ABLF controller with SOC control layer significantly reduces the steady-state error of the system compared to the other controllers evaluated, with a small settling time (15ms) and an overshoot less than 2%. Regarding the settling time and overshoot, the backstepping ABLF controller optimized offline with SLDO has a considerable bigger settling time with almost no overshoot. In the case of the ABLF controller without SOC, it has the biggest overshoot around 8% and the smallest settling time (13ms).

Thus, we can say that the backstepping ABLF controller with an SOC layer is able to improve the position tracking response of the non-linear system, compensating for the steady state error, overshoot, and settling time as well as reducing the effect of model disturbances and uncertainties within a reasonable time while monitoring the closed-loop response of the system.

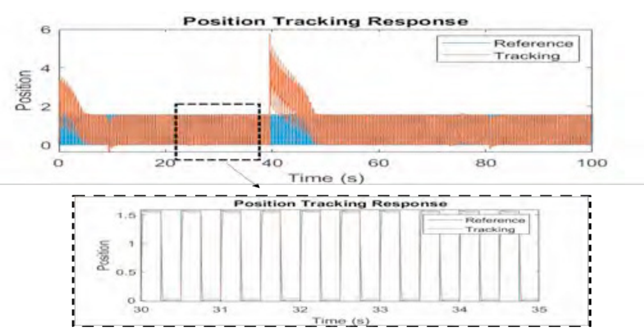

Fig. 2. Backstepping controller design response before optimization. Top - from 0 to 100s, bottom - from 30 to 35s

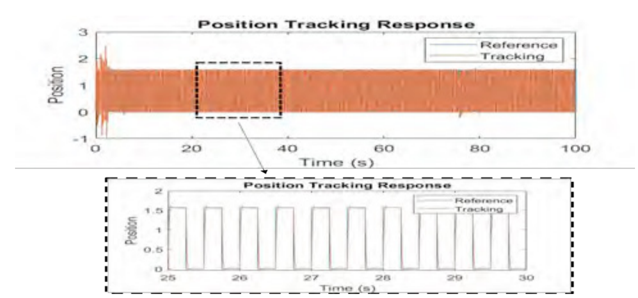

Fig. 3. Backstepping controller design response after optimization. Top - from 0 to 100s, bottom - from 30 to 35s

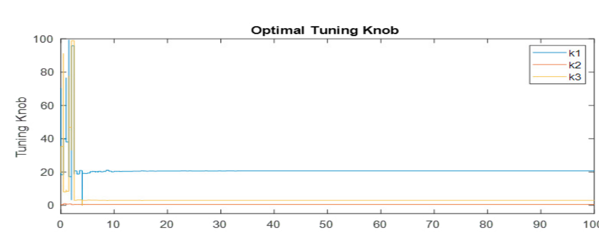

Fig. 4. Optimal tuning gains

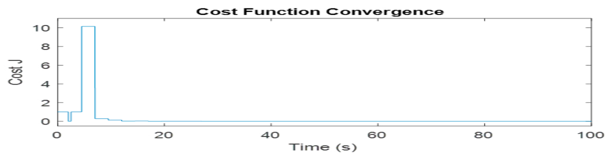

Fig. 5. Cost function convergence

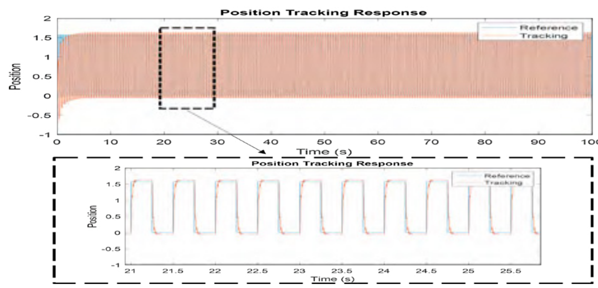

Fig. 6. Backstepping controller design response after optimization with SLDO. Top - from 0 to 100s, bottom - from 25 to 30s

Table 1. Backstepping ABLF Controllers Performance Assessment

Parameter	Backstepping Controller		
	ABLF	ABLF SOC	ABLF SLDO
k_1	10	2.931	6.2718
k_2	1	0.474	2399
k_3	10	20.693	31.9167
Overshoot (%)	8.03	1.9	0.35
Settling time (ms)	13	15.7	100
Error	0.047	0.012	0.065

8. CONCLUSIONS AND FUTURE WORK

In this paper, a backstepping ABLF dynamic surface control (DSC) with an observer-based friction torque compensator is combined with a self optimizing control (SOC) layer for the position tracking control of a non-linear brushless DC (BLDC) motor system. The SOC controller employed the Globalized constrained optimization algorithm (GCNM) for the online monitoring of the closed-loop system performance employing the steady-state error as cost function.

The obtained results show that the performance of the backstepping ABLF controller is not robust enough for a prolonged execution of the system, producing disturbances and uncertainties on the system which affect the tracking position performance. If an offline optimization procedure of the controller parameters is performed, it leads to high gains that makes the system more sensitive to random noise, also the controller is not able to compensate properly for the steady-state error.

For this reason, the SOC layer is included to provide a supervisory layer that monitors the closed-loop performance and adjusts the controller terms to keep the position tracking under unexpected disturbances and uncertainties of the model. Thus, the SOC control architecture presented in this paper can be applied to other nonlinear systems with variable time behavior.

As future works, a second SOC layer can be added to monitor and optimize the friction constant estimation a. Moreover, a high order online optimization problem can be solved by introducing more tuning knobs like τ_2 and τ_3. However, a rigorous theoretical convergence, stability, and globalness analysis of the SOC controller with nonlinear systems are required to ensure the algorithm convergence in finite time.

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